Numerical and Statistical Quantifications of Biodiversity: Two-At-A-Time Equal Variations

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Abstract— *The ecological concept of biodiversity is a challenging environmental problem that requires a sound mathematical reasoning. We have used the method of a numerical simulation that is indexed by a numerical scheme to predict biodiversity loss and biodiversity gain due to a decreasing and increasing variations of the intrinsic growth rates together. The novel results that we have obtained that we have not seen elsewhere, but do complement other similar numerical predictions of biodiversity are presented and discussed quantitatively.*

***Keywords—* Ecological concept, environmental problem, biodiversity, numerical simulation, intrinsic growth rate, ecosystem stability.**

I. INTRODUCTION

The ongoing debate between biodiversity, ecosystem stability, and its implications, Atsu and Ekaka-a(2017)[1] makes it imperative to examine the effects of varying the intrinsic growth rates together on biodiversity loss and biodiversity gain by using a computationally efficient numerical scheme called Matlab function ordinary differential equation of order 45 (ODE 45). Other related contributions on the link between biodiversity and ecosystem stability have been adequately sighted. ([2] – [26]).

II. MATERIALS AND METHODS

If a variation of a model parameter value produces a new biomass which is smaller than the old biomass for any interacting legumes, such as cowpea and groundnut, then a biodiversity loss has occurred and can be quantified as we have done in this study.

On the other hand, if a variation of a model parameter value produces a new biomass which outweighs the old biomass irrespective of the type of legumes, then a biodiversity gain has occurred and can be similarly quantified.

Following Ekaka-a et al (2009), we have considered the following continuous dynamical system of nonlinear first order ordinary differential equation

$$\frac{d}{dt}C(t) = \alpha_1 C(t) - \beta_1 C^2(t) - r_1 C(t)G(t)$$

$$\frac{d}{dt}G(t) = \alpha_2 G(t) - \beta_2 G^2(t) - r_2 C(t)G(t)$$

With $C(0) = 0.12$ and $G(0) = 0.14$

For the purpose of clarity, the variables and the parameter values for these model equations are defined as follows

- $C_b(t)$ and $G_b(t)$ are called the biomass of cowpea and groundnut at time (t) in the unit of weeks
- α_1 and α_2 are called the intrinsic growth rates for populations $C_b(t)$ and $G_b(t)$ in the absence of self-interaction and inter-competition interaction
- β_1 and β_2 are called the intra-competition coefficients
- r_1 and r_2 are called the inter-competition coefficients to analyze our propose problem,

$\alpha_1 = 0.0225, \alpha_2 = 0.0446$; $\beta_1 = 0.0069$, $\beta_2 = 0.0133$; $r_1 = 0.0018$, $r_2 = 0.0012$.

The core numerical method that we have used in this present analysis is called ODE 45.

III. RESULTS AND DISCUSSIONS

The results of this study are displayed and discussed quantitatively in Tables 1.1, 1.2, 1.3, 1.4, 1.5, 2.1, 2.2, 2.3, 2.4 and 2.5.

Table.1.1: Evaluating the effect of varying the intrinsic growth rates together by 10% on biodiversity loss using ODE 45 numerical scheme.

$C_b(t)$	$C_{bm}(t)$	BL(%)	$G_b(t)$	$G_{bm}(t)$	BL(%)
0.1200	0.1200	0	0.1200	0.1200	0
0.1226	0.1201	2.0034	0.1253	0.1203	3.9312
0.1253	0.1203	3.9641	0.1307	0.1207	7.7015
0.1280	0.1204	5.8831	0.1364	0.1210	11.3173
0.1307	0.1206	7.7611	0.1424	0.1213	14.7850
0.1335	0.1207	9.5991	0.1485	0.1216	18.1105
0.1364	0.1209	11.3979	0.1550	0.1220	21.2998
0.1393	0.1210	13.1582	0.1617	0.1223	24.3584
0.1423	0.1212	14.8809	0.1686	0.1226	27.2915

0.1454	0.1213	16.5668	0.1759	0.1230	30.1044
0.1485	0.1214	18.2166	0.1835	0.1233	32.8018
0.1517	0.1216	19.8310	0.1913	0.1236	35.3886
0.1549	0.1217	21.4108	0.1995	0.1239	37.8693
0.1582	0.1219	22.9568	0.2080	0.1243	40.2481
0.1616	0.1220	24.4695	0.2168	0.1246	42.5293
0.1650	0.1222	25.9497	0.2260	0.1249	44.7168
0.1685	0.1223	27.3980	0.2355	0.1253	46.8144
0.1720	0.1225	28.8152	0.2454	0.1256	48.8258
0.1757	0.1226	30.2018	0.2557	0.1259	50.7546
0.1793	0.1227	31.5585	0.2664	0.1263	52.6041

From Table 1.1, when all the model parameter values are fixed, the cowpea biomass data denoted $C_b(t)$ when the length of the growing season is twenty one weeks range from a low value of 0.12grams/area to 0.1793grams/area whereas $C_{bm}(t)$ data range from a low value 0.12 grams/area to 0.1227 grams/area due to a 10% variation of the intrinsic growth rates together. On the basis of this calculation, the new simulated cowpea data due to a joint variation of the intrinsic growth rates dominantly predicts a depletion which mimics biodiversity loss. The extent of biodiversity loss has been quantified to range from zero to 31.6 approximately providing an average of 16.8 which re-classifies the vulnerability of the cowpea biomass to biodiversity loss. A similar observation can be made from the groundnut biomass component. In summary, the groundnut biomass is about 1.67 approximately more vulnerable to biodiversity loss than the cowpea biomass. Statistically, the average of biomass vulnerability to biodiversity loss with respect to the groundnut legume is 29.57% approximately.

Table.1.2: Evaluating the effect of varying the intrinsic growth rates together by 15% on biodiversity loss using ODE 45 numerical scheme.

$C_b(t)$	$C_{bm}(t)$	BL(%)	$G_b(t)$	$G_{bm}(t)$	BL(%)
0.1200	0.1200	0	0.1200	0.1200	0
0.1226	0.1203	1.8931	0.1253	0.1206	3.7169
0.1253	0.1206	3.7480	0.1307	0.1212	7.2896
0.1280	0.1208	5.5655	0.1364	0.1218	10.7235
0.1307	0.1211	7.3462	0.1424	0.1224	14.0240
0.1335	0.1214	9.0908	0.1485	0.1230	17.1963
0.1364	0.1217	10.8001	0.1550	0.1236	20.2451
0.1393	0.1220	12.4747	0.1617	0.1242	23.1754
0.1423	0.1222	14.1153	0.1686	0.1248	25.9917
0.1454	0.1225	15.7226	0.1759	0.1254	28.6983
0.1485	0.1228	17.2972	0.1835	0.1260	31.2995
0.1517	0.1231	18.8398	0.1913	0.1267	33.7993
0.1549	0.1234	20.3509	0.1995	0.1273	36.2017
0.1582	0.1237	21.8312	0.2080	0.1279	38.5104
0.1616	0.1239	23.2813	0.2168	0.1285	40.7289
0.1650	0.1242	24.7017	0.2260	0.1291	42.8609
0.1685	0.1245	26.0930	0.2355	0.1298	44.9095
0.1720	0.1248	27.4558	0.2454	0.1304	46.8781
0.1757	0.1251	28.7906	0.2557	0.1310	48.7696
0.1793	0.1254	30.0981	0.2664	0.1316	50.5872

From Table 1.2, when all the model parameter values are fixed, the cowpea biomass data denoted $C_b(t)$ when the length of the growing season is twenty one weeks range from a low value of 0.12 grams/area to 0.1793 grams/area whereas $C_{bm}(t)$ data range from a low value 0.12 grams/area to 0.1254 grams/area due to a 15% variation of the intrinsic growth rates together. On the basis of this calculation, the new simulated cowpea data due to a joint variation of the intrinsic growth rates dominantly predicts a depletion which mimics biodiversity loss. The extent of biodiversity loss has been quantified to range from zero to 30.1 approximately providing an average of 15.97 which re-classifies the vulnerability of the cowpea biomass to biodiversity loss. A similar observation can be made from the groundnut biomass component. In summary, the groundnut biomass is about 1.68 approximately more vulnerable to biodiversity loss than the cowpea biomass. Statistically, the average of biomass vulnerability to biodiversity loss with respect to the groundnut legume is 28.28% approximately.

Table.1.3: Evaluating the effect of varying the intrinsic growth rates together by 20% on biodiversity loss using ODE 45 numerical scheme.

$C_b(t)$	$C_{bm}(t)$	BL(%)	$G_b(t)$	$G_{bm}(t)$	BL(%)
0.1200	0.1200	0	0.1200	0.1200	0
0.1226	0.1204	1.7828	0.1253	0.1209	3.5022
0.1253	0.1208	3.5315	0.1307	0.1217	6.8759
0.1280	0.1212	5.2468	0.1364	0.1226	10.1258
0.1307	0.1217	6.9293	0.1424	0.1235	13.2563
0.1335	0.1221	8.5796	0.1485	0.1244	16.2718
0.1364	0.1225	10.1982	0.1550	0.1253	19.1764
0.1393	0.1229	11.7858	0.1617	0.1261	21.9741
0.1423	0.1233	13.3429	0.1686	0.1270	24.6688
0.1454	0.1238	14.8699	0.1759	0.1279	27.2641
0.1485	0.1242	16.3676	0.1835	0.1289	29.7638

0.1517	0.1246	17.8364	0.1913	0.1298	32.1712
0.1549	0.1250	19.2768	0.1995	0.1307	34.4897
0.1582	0.1255	20.6893	0.2080	0.1316	36.7225
0.1616	0.1259	22.0745	0.2168	0.1325	38.8727
0.1650	0.1263	23.4328	0.2260	0.1335	40.9432
0.1685	0.1267	24.7647	0.2355	0.1344	42.9371
0.1720	0.1272	26.0707	0.2454	0.1353	44.8570
0.1757	0.1276	27.3513	0.2557	0.1363	46.7055
0.1793	0.1280	28.6068	0.2664	0.1372	48.4854

From Table 1.3, when all the model parameter values are fixed, the cowpea biomass data $C_b(t)$ when the length of the growing season is twenty one weeks range from a low value of 0.12 grams/area to 0.1793 grams/area whereas $C_{bm}(t)$ data range from a low value 0.12 grams/area to 0.1280 grams/area due to a 20% variation of the intrinsic growth rates together. On the basis of this calculation, the new simulated cowpea data due to a joint variation of the intrinsic growth rates dominantly predicts a depletion which mimics biodiversity loss. The extent of biodiversity loss has been quantified to range from zero to 28.6 approximately providing an average of 15.14 which re-classifies the vulnerability of the cowpea biomass to biodiversity loss. A similar observation can be made from the groundnut biomass component. In summary, the groundnut biomass is about 1.69 approximately more vulnerable to biodiversity loss than the cowpea biomass. Statistically, the average of biomass vulnerability to biodiversity loss with respect to the groundnut legume is 26.95% approximately.

Table.1.4: Evaluating the effect of varying the intrinsic growth rates together by 25% on biodiversity loss using ODE 45 numerical scheme.

$C_b(t)$	$C_{bm}(t)$	BL(%)	$G_b(t)$	$G_{bm}(t)$	BL(%)
0.1200	0.1200	0	0.1200	0.1200	0
0.1226	0.1206	1.6723	0.1253	0.1211	3.2869
0.1253	0.1211	3.3145	0.1307	0.1223	6.4603
0.1280	0.1217	4.9271	0.1364	0.1234	9.5240
0.1307	0.1222	6.5106	0.1424	0.1246	12.4818
0.1335	0.1228	8.0656	0.1485	0.1258	15.3370
0.1364	0.1233	9.5924	0.1550	0.1269	18.0933
0.1393	0.1239	11.0915	0.1617	0.1281	20.7540
0.1423	0.1245	12.5635	0.1686	0.1293	23.3223
0.1454	0.1250	14.0087	0.1759	0.1305	25.8013
0.1485	0.1256	15.4276	0.1835	0.1317	28.1940
0.1517	0.1262	16.8207	0.1913	0.1330	30.5033
0.1549	0.1267	18.1883	0.1995	0.1342	32.7321
0.1582	0.1273	19.5309	0.2080	0.1354	34.8830
0.1616	0.1279	20.8489	0.2168	0.1367	36.9588
0.1650	0.1284	22.1427	0.2260	0.1379	38.9619
0.1685	0.1290	23.4128	0.2355	0.1392	40.8947
0.1720	0.1296	24.6594	0.2454	0.1405	42.7597
0.1757	0.1302	25.8831	0.2557	0.1418	44.5592
0.1793	0.1308	27.0841	0.2664	0.1431	46.2954

Table.1.5: Evaluating the effect of varying the intrinsic growth rates together by 95% on biodiversity loss using ODE 45 numerical scheme.

$C_b(t)$	$C_{bm}(t)$	BL(%)	$G_b(t)$	$G_{bm}(t)$	BL(%)
0.1200	0.1200	0	0.1200	0.1200	0
0.1226	0.1225	0.1124	0.1253	0.1250	0.2226
0.1253	0.1250	0.2245	0.1307	0.1301	0.4442
0.1280	0.1275	0.3363	0.1364	0.1355	0.6650
0.1307	0.1301	0.4478	0.1424	0.1411	0.8847
0.1335	0.1328	0.5590	0.1485	0.1469	1.1035
0.1364	0.1355	0.6699	0.1550	0.1529	1.3213
0.1393	0.1383	0.7805	0.1617	0.1592	1.5381
0.1423	0.1411	0.8908	0.1686	0.1657	1.7537
0.1454	0.1439	1.0007	0.1759	0.1724	1.9682
0.1485	0.1468	1.1103	0.1835	0.1795	2.1815
0.1517	0.1498	1.2195	0.1913	0.1867	2.3936
0.1549	0.1528	1.3284	0.1995	0.1943	2.6045
0.1582	0.1559	1.4369	0.2080	0.2021	2.8141
0.1616	0.1591	1.5450	0.2168	0.2103	3.0223
0.1650	0.1623	1.6527	0.2260	0.2187	3.2291
0.1685	0.1655	1.7600	0.2355	0.2274	3.4344
0.1720	0.1688	1.8668	0.2454	0.2365	3.6383
0.1757	0.1722	1.9733	0.2557	0.2459	3.8406
0.1793	0.1756	2.0792	0.2664	0.2556	4.0412

Table.2.1: Evaluating the effect of varying the intrinsic growth rates together by 105% on biodiversity gain using ODE 45 numerical scheme.

$C_b(t)$	$C_{bm}(t)$	BG(%)	$G_b(t)$	$G_{bm}(t)$	BG(%)
0.1200	0.1200	0	0.1200	0.1200	0
0.1226	0.1227	0.1125	0.1253	0.1255	0.2231
0.1253	0.1255	0.2250	0.1307	0.1313	0.4462
0.1280	0.1284	0.3374	0.1364	0.1373	0.6694
0.1307	0.1313	0.4498	0.1424	0.1436	0.8926
0.1335	0.1343	0.5621	0.1485	0.1502	1.1158
0.1364	0.1373	0.6744	0.1550	0.1570	1.3389

0.1393	0.1404	0.7866	0.1617	0.1642	1.5620
0.1423	0.1436	0.8987	0.1686	0.1717	1.7848
0.1454	0.1469	1.0108	0.1759	0.1794	2.0074
0.1485	0.1502	1.1227	0.1835	0.1875	2.2298
0.1517	0.1535	1.2345	0.1913	0.1960	2.4518
0.1549	0.1570	1.3461	0.1995	0.2048	2.6735
0.1582	0.1605	1.4576	0.2080	0.2140	2.8947
0.1616	0.1641	1.5690	0.2168	0.2236	3.1153
0.1650	0.1678	1.6801	0.2260	0.2335	3.3354
0.1685	0.1715	1.7911	0.2355	0.2439	3.5548
0.1720	0.1753	1.9018	0.2454	0.2547	3.7734
0.1757	0.1792	2.0124	0.2557	0.2659	3.9912
0.1793	0.1832	2.1226	0.2664	0.2776	4.2080

Table.2.2: Evaluating the effect of varying the intrinsic growth rates together by 110% on biodiversity gain using ODE 45 numerical scheme.

$C_b(t)$	$C_{bm}(t)$	BG(%)	$G_b(t)$	$G_{bm}(t)$	BG(%)
0.1200	0.1200	0	0.1200	0.1200	0
0.1226	0.1229	0.2251	0.1253	0.1258	0.4466
0.1253	0.1258	0.4504	0.1307	0.1319	0.8944
0.1280	0.1288	0.6759	0.1364	0.1383	1.3433
0.1307	0.1319	0.9016	0.1424	0.1449	1.7932
0.1335	0.1350	1.1274	0.1485	0.1519	2.2441
0.1364	0.1383	1.3533	0.1550	0.1592	2.6958
0.1393	0.1415	1.5794	0.1617	0.1668	3.1482
0.1423	0.1449	1.8055	0.1686	0.1747	3.6013
0.1454	0.1483	2.0317	0.1759	0.1830	4.0549
0.1485	0.1518	2.2578	0.1835	0.1917	4.5089
0.1517	0.1554	2.4840	0.1913	0.2008	4.9632
0.1549	0.1591	2.7102	0.1995	0.2103	5.4177
0.1582	0.1628	2.9363	0.2080	0.2202	5.8722
0.1616	0.1667	3.1623	0.2168	0.2305	6.3265
0.1650	0.1706	3.3881	0.2260	0.2413	6.7804
0.1685	0.1746	3.6138	0.2355	0.2526	7.2339
0.1720	0.1786	3.8393	0.2454	0.2643	7.6867
0.1757	0.1828	4.0646	0.2557	0.2765	8.1387
0.1793	0.1870	4.2895	0.2664	0.2893	8.5895

Table.2.3: Evaluating the effect of varying the intrinsic growth rates together by 115% on biodiversity gain using ODE 45 numerical scheme.

$C_b(t)$	$C_{bm}(t)$	BG(%)	$G_b(t)$	$G_{bm}(t)$	BG(%)
0.1200	0.1200	0	0.1200	0.1200	0
0.1226	0.1230	0.3379	0.1253	0.1261	0.6707
0.1253	0.1261	0.6764	0.1307	0.1325	1.3446
0.1280	0.1293	1.0156	0.1364	0.1392	2.0217
0.1307	0.1325	1.3554	0.1424	0.1462	2.7018
0.1335	0.1358	1.6959	0.1485	0.1536	3.3848
0.1364	0.1392	2.0368	0.1550	0.1613	4.0707
0.1393	0.1427	2.3784	0.1617	0.1694	4.7591
0.1423	0.1462	2.7204	0.1686	0.1778	5.4500
0.1454	0.1498	3.0628	0.1759	0.1867	6.1432
0.1485	0.1536	3.4057	0.1835	0.1960	6.8385
0.1517	0.1574	3.7489	0.1913	0.2057	7.5357
0.1549	0.1612	4.0924	0.1995	0.2159	8.2345
0.1582	0.1652	4.4362	0.2080	0.2266	8.9348
0.1616	0.1693	4.7803	0.2168	0.2377	9.6363
0.1650	0.1734	5.1244	0.2260	0.2494	10.3387
0.1685	0.1777	5.4687	0.2355	0.2615	11.0417
0.1720	0.1820	5.8130	0.2454	0.2743	11.7451
0.1757	0.1865	6.1573	0.2557	0.2875	12.4484
0.1793	0.1910	6.5016	0.2664	0.3014	13.1514

Table.2.4: Evaluating the effect of varying the intrinsic growth rates together by 120% on biodiversity gain using ODE 45 numerical scheme.

$C_b(t)$	$C_{bm}(t)$	BG(%)	$G_b(t)$	$G_{bm}(t)$	BG(%)
0.1200	0.1200	0	0.1200	0.1200	0
0.1226	0.1232	0.4507	0.1253	0.1264	0.8952
0.1253	0.1264	0.9029	0.1307	0.1331	1.7968
0.1280	0.1297	1.3564	0.1364	0.1401	2.7046
0.1307	0.1331	1.8113	0.1424	0.1475	3.6185
0.1335	0.1366	2.2675	0.1485	0.1553	4.5383
0.1364	0.1401	2.7249	0.1550	0.1634	5.4639
0.1393	0.1438	3.1836	0.1617	0.1720	6.3950
0.1423	0.1475	3.6434	0.1686	0.1810	7.3315
0.1454	0.1514	4.1043	0.1759	0.1905	8.2731
0.1485	0.1553	4.5663	0.1835	0.2004	9.2195
0.1517	0.1593	5.0292	0.1913	0.2108	10.1706
0.1549	0.1634	5.4931	0.1995	0.2217	11.1258
0.1582	0.1676	5.9578	0.2080	0.2331	12.0850
0.1616	0.1719	6.4233	0.2168	0.2451	13.0478

0.1650	0.1763	6.8895	0.2260	0.2577	14.0138
0.1685	0.1809	7.3564	0.2355	0.2708	14.9825
0.1720	0.1855	7.8237	0.2454	0.2846	15.9536
0.1757	0.1902	8.2915	0.2557	0.2990	16.9265
0.1793	0.1951	8.7597	0.2664	0.3141	17.9007

Table.2.5: Evaluating the effect of varying the intrinsic growth rates together by 125% on biodiversity gain using ODE 45 numerical scheme.

$C_b(t)$	$C_{bm}(t)$	BG(%)	$G_b(t)$	$G_{bm}(t)$	BG(%)
0.1200	0.1200	0	0.1200	0.1200	0
0.1226	0.1233	0.5637	0.1253	0.1267	1.1203
0.1253	0.1267	1.1299	0.1307	0.1337	2.2510
0.1280	0.1301	1.6984	0.1364	0.1410	3.3921
0.1307	0.1337	2.2692	0.1424	0.1488	4.5433
0.1335	0.1373	2.8423	0.1485	0.1570	5.7046
0.1364	0.1411	3.4176	0.1550	0.1656	6.8757
0.1393	0.1449	3.9951	0.1617	0.1747	8.0564
0.1423	0.1488	4.5747	0.1686	0.1842	9.2464
0.1454	0.1529	5.1563	0.1759	0.1943	10.4454
0.1485	0.1570	5.7398	0.1835	0.2048	11.6532
0.1517	0.1613	6.3253	0.1913	0.2159	12.8694
0.1549	0.1656	6.9125	0.1995	0.2276	14.0935
0.1582	0.1701	7.5014	0.2080	0.2399	15.3253
0.1616	0.1746	8.0919	0.2168	0.2527	16.5641
0.1650	0.1793	8.6839	0.2260	0.2662	17.8095
0.1685	0.1841	9.2773	0.2355	0.2804	19.0609
0.1720	0.1890	9.8720	0.2454	0.2953	20.3177
0.1757	0.1940	10.4678	0.2557	0.3109	21.5792
0.1793	0.1992	11.0647	0.2664	0.3273	22.8448

Statistical measure by Table	BL_1 (Average)	BL_2 (Average)
Table 1.1	16.801	29.5726
Table 1.2	15.9748	28.2803
Table 1.3	15.1369	26.9532
Table 1.4	14.2872	25.5902
Table 1.5	1.0497	2.0550
Statistical measure by Table	BG_1 (Average)	BG_1 (Average)
Table 2.1	1.0648	2.1134
Table 2.2	2.1448	4.2870
Table 2.3	3.2404	6.5226
Table 2.4	4.3518	8.8221
Table 2.5	5.4792	11.1876

IV. CONCLUSION

By using ODE 45 we have found out that a biodiversity loss can be obtained due to a decreasing variation of the intrinsic growth rates together, whereas a dominant biodiversity gain can be obtained due to an increasing variation of the intrinsic growth rates together. On the basis of this analysis, the decreasing variation of the intrinsic growth rates together has generally indicated a decrease in the yields of these two crops, whereas an increasing variation of the same parameter values has indicated an improvement in the yields of both cowpea and groundnut. In this context, an alarming rate of biodiversity loss of these quantified magnitude are a strong signal on lower food production, endemic poverty and a weak sustainable development scenario, whereas a biodiversity gain has the potential to alleviate poverty and sustain development. These two components of biodiversity as predicted in this work have their policy implications.

This present numerical idea can be extended to examine the effects of varying the intra and inter competition coefficients together in our future investigation.

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