Simulation modelling of the effect of a random disturbance on biodiversity of a mathematical model of mutualism between two interacting yeast species

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Abstract— *The effect of a random disturbance on the ecosystem is one of the oldest scientific observations of which its effect on biodiversity is no exception. We have used ODE 45 numerical scheme to tackle this problem. The novel results that we have obtained have not been seen elsewhere; these are presented and fully discussed quantitatively.*

Keywords— Random disturbance, numerical scheme, biodiversity, dynamical system, stochastic, deterministic dynamical system.

I. INTRODUCTION

An ecological dynamical system is inherently stochastic in its scientific construction and definition. In this scenario, a deterministic definition of an ecological dynamical system is a special case of a stochastic ecological system that is more highly vulnerable to random disturbance which can be attributed to the other environmental and climatic factors and other characteristics of the ecosystem which we cannot go into in detailed discussion. However, there are two factors that may have a high potential to influence the performance of biodiversity gain. One of these factors could be a conducive steady environment that is less hostile to interaction between yeast populations. The other factor could be attributed to an ecological system where human activities do not have a huge impact on the growing yeast species. These two factors put together are capable to improve the performance of yeast species in terms of their yields that can mimic strong evidence of biodiversity gain. In other words, a random noise disturbance in terms of these mentioned factors may not necessarily bring about biodiversity loss but are capable to increase the magnitude of biodiversity gain.

II. MATERIALS AND METHODS

We have considered a semi – stochastic fashion of our deterministic dynamical system in which a dynamical system with two random noise perturbation scenarios of 0.01 and 0.1 in the first instance and next for a random noise perturbation of 0.8. This method is based on the 150 percent variation of the inter-competition coefficients together.

III. RESULTS

The corresponding results of this study are presented in Table 1, Table 2, Table 3, Table 4, Table 5, and Table 6

Table.1: Quantifying the effect of a random disturbance having the intensity of 0.01 on biodiversity gain using ODE 45 numerical scheme. Scenario One

Example	$x(t)$	$x_m(t)$	BG (%)	$y(t)$	$y_m(t)$	BG (%)
1	4.0000	4.0000	0	10.0000	10.0000	0
2	4.4497	4.5132	1.4276	10.7618	10.8509	0.8273
3	4.9514	5.1133	3.2706	11.5776	11.7320	1.3340
4	5.5111	5.7691	4.6831	12.4505	12.6952	1.9650
5	6.1356	6.5317	6.4556	13.3844	13.7425	2.6755
6	6.8325	7.3519	7.6024	14.3829	14.8480	3.2337
7	7.6102	8.2747	8.7320	15.4502	16.0596	3.9444
8	8.4778	9.3598	10.4040	16.5906	17.3229	4.4139

Example	$x(t)$	$x_m(t)$	BG (%)	$y(t)$	$y_m(t)$	BG (%)
9	9.4456	10.6178	12.4105	17.8090	18.7357	5.2036
10	10.5247	12.0170	14.1789	19.1103	20.2260	5.8379
11	11.7273	13.5941	15.9187	20.5002	21.8492	6.5805
12	13.0666	15.3911	17.7895	21.9847	23.6066	7.3774
13	14.5569	17.4257	19.7075	23.5705	25.5991	8.6065
14	16.2134	19.7543	21.8394	25.2650	27.7541	9.8517
15	18.0522	22.4048	24.1112	27.0765	30.0533	10.9939
16	20.0902	25.4003	26.4313	29.0141	32.5986	12.3545
17	22.3450	28.8608	29.1598	31.0880	35.4470	14.0217
18	24.8344	32.6905	31.6343	33.3096	38.6581	16.0567
19	27.5763	37.0962	34.5221	35.6920	42.2245	18.3025
20	30.5884	42.0707	37.5382	38.2492	46.2160	20.8286

Table.2: Quantifying the effect of a random disturbance having the intensity of 0.01 on biodiversity gain using ODE 45 numerical scheme. Scenario Two

Example	$x(t)$	$x_m(t)$	BG(%)	$y(t)$	$y_m(t)$	BG(%)
1	4.0000	4.0000	0	10.0000	10.0000	0
2	4.4497	4.5237	1.6635	10.7618	10.8162	0.5054
3	4.9514	5.1350	3.7100	11.5776	11.7026	1.0797
4	5.5111	5.8552	6.2449	12.4505	12.7082	2.0699
5	6.1356	6.6134	7.7883	13.3844	13.7418	2.6703
6	6.8325	7.5144	9.9797	14.3829	14.9599	4.0116
7	7.6102	8.4799	11.4283	15.4502	16.2132	4.9386
8	8.4778	9.5569	12.7289	16.5906	17.5598	5.8418
9	9.4456	10.8277	14.6327	17.8090	19.0138	6.7651
10	10.5247	12.2314	16.2166	19.1103	20.5377	7.4689
11	11.7273	13.8468	18.0734	20.5002	22.2450	8.5111
12	13.0666	15.6921	20.0927	21.9847	24.0618	9.4480
13	14.5569	17.7736	22.0973	23.5705	26.0330	10.4475
14	16.2134	20.1278	24.1427	25.2650	28.1513	11.4237
15	18.0522	22.7965	26.2811	27.0765	30.5581	12.8584
16	20.0902	25.8497	28.6678	29.0141	33.2120	14.4686
17	22.3450	29.3634	31.4092	31.0880	36.0546	15.9761
18	24.8344	33.3098	34.1278	33.3096	39.2617	17.8690
19	27.5763	37.7921	37.0457	35.6920	42.9179	20.2453
20	30.5884	42.8728	40.1607	38.2492	46.9771	22.8184

Table.3: Quantifying the effect of a random disturbance having the intensity of 0.1 on biodiversity gain using ODE 45 numerical scheme. Scenario Three

Example	$x(t)$	$x_m(t)$	BG(%)	$y(t)$	$y_m(t)$	BG(%)
1	4.0000	4.0000	0	10.0000	10.0000	0
2	4.4497	4.5334	1.8803	10.7618	10.8330	0.6613
3	4.9514	5.0793	2.5839	11.5776	11.7789	1.7395
4	5.5111	5.7124	3.6543	12.4505	12.7405	2.3287
5	6.1356	6.4569	5.2377	13.3844	13.7864	3.0037
6	6.8325	7.2961	6.7848	14.3829	14.9548	3.9766
7	7.6102	8.2988	9.0489	15.4502	16.1720	4.6722
8	8.4778	9.3920	10.7836	16.5906	17.4731	5.3191
9	9.4456	10.6431	12.6781	17.8090	18.9168	6.2202
10	10.5247	12.0703	14.6858	19.1103	20.4365	6.9397

Example	$x(t)$	$x_m(t)$	$BG(\%)$	$y(t)$	$y_m(t)$	$BG(\%)$
11	11.7273	13.6683	16.5511	20.5002	22.1251	7.9260
12	13.0666	15.4880	18.5308	21.9847	23.9678	9.0203
13	14.5569	17.4978	20.2024	23.5705	25.9385	10.0464
14	16.2134	19.8987	22.7301	25.2650	28.1712	11.5028
15	18.0522	22.5636	24.9908	27.0765	30.5360	12.7766
16	20.0902	25.5495	27.1740	29.0141	33.1085	14.1121
17	22.3450	28.9518	29.5672	31.0880	36.0206	15.8668
18	24.8344	32.8223	32.1648	33.3096	39.2187	17.7398
19	27.5763	37.2030	34.9093	35.6920	42.7775	19.8519
20	30.5884	42.1760	37.8825	38.2492	46.7649	22.2637

Table.4: Quantifying the effect of a random disturbance having the intensity of 0.1 on biodiversity gain using ODE 45 numerical scheme. Scenario Four

Example	$x(t)$	$x_m(t)$	$BG(\%)$	$y(t)$	$y_m(t)$	$BG(\%)$
1	4.0000	4.0000	0	10.0000	10.0000	0
2	4.4497	4.4867	0.8312	10.7618	10.8039	0.3907
3	4.9514	5.0647	2.2898	11.5776	11.6835	0.9150
4	5.5111	5.6951	3.3405	12.4505	12.6824	1.8624
5	6.1356	6.5067	6.0488	13.3844	13.7450	2.6946
6	6.8325	7.3949	8.2316	14.3829	14.8654	3.3550
7	7.6102	8.3792	10.1059	15.4502	16.0553	3.9163
8	8.4778	9.4514	11.4848	16.5906	17.3531	4.5954
9	9.4456	10.7237	13.5309	17.8090	18.7569	5.3224
10	10.5247	12.1772	15.7008	19.1103	20.2479	5.9526
11	11.7273	13.7391	17.1551	20.5002	21.9514	7.0789
12	13.0666	15.5557	19.0486	21.9847	23.7044	7.8224
13	14.5569	17.6162	21.0155	23.5705	25.6845	8.9686
14	16.2134	19.9303	22.9250	25.2650	27.9186	10.5029
15	18.0522	22.5407	24.8641	27.0765	30.2741	11.8094
16	20.0902	25.5231	27.0421	29.0141	32.9389	13.5275
17	22.3450	28.9112	29.3857	31.0880	35.7840	15.1058
18	24.8344	32.8304	32.1974	33.3096	38.9832	17.0329
19	27.5763	37.2158	34.9561	35.6920	42.5658	19.2588
20	30.5884	42.1994	37.9592	38.2492	46.5714	21.7578

Table.5: Quantifying the effect of a random disturbance having the intensity of 0.8 on biodiversity gain using ODE 45 numerical scheme. Scenario Five

Example	$x(t)$	$x_m(t)$	$BG(\%)$	$y(t)$	$y_m(t)$	$BG(\%)$
1	4.0000	4.0000	0	10.0000	10.0000	0
2	4.4497	4.8324	8.6014	10.7618	11.4308	6.2161
3	4.9514	5.8134	17.4109	11.5776	12.7359	10.0048
4	5.5111	6.9156	25.4863	12.4505	14.2481	14.4378
5	6.1356	8.2866	35.0587	13.3844	15.5423	16.1229
6	6.8325	9.6980	41.9400	14.3829	17.1956	19.5561
7	7.6102	11.2228	47.4713	15.4502	18.8135	21.7691
8	8.4778	12.9207	52.4067	16.5906	20.7384	25.0008
9	9.4456	15.1202	60.0765	17.8090	22.8115	28.0896
10	10.5247	17.3165	64.5317	19.1103	25.0589	31.1277
11	11.7273	19.9405	70.0344	20.5002	27.5825	34.5477
12	13.0666	22.7270	73.9314	21.9847	30.4937	38.7040

Example	$x(t)$	$x_m(t)$	$BG(\%)$	$y(t)$	$y_m(t)$	$BG(\%)$
13	14.5569	26.0227	78.7651	23.5705	33.5233	42.2257
14	16.2134	29.7098	83.2424	25.2650	36.7004	45.2614
15	18.0522	34.0788	88.7790	27.0765	40.6045	49.9621
16	20.0902	39.0831	94.5379	29.0141	44.6849	54.0111
17	22.3450	44.9096	100.9828	31.0880	48.9859	57.5719
18	24.8344	51.3077	106.5995	33.3096	53.9942	62.0979
19	27.5763	58.3908	111.7429	35.6920	60.0885	68.3532
20	30.5884	66.7729	118.2950	38.2492	66.8022	74.6498

Table.6: Quantifying the effect of a random disturbance having the intensity of 0.8 on biodiversity gain using ODE 45 numerical scheme. Scenario Six

Example	$x(t)$	$x_m(t)$	$BG(\%)$	$y(t)$	$y_m(t)$	$BG(\%)$
1	4.0000	4.0000	0	10.0000	10.0000	0
2	4.4497	4.8883	9.8575	10.7618	11.1320	3.4394
3	4.9514	5.9583	20.3360	11.5776	12.4481	7.5191
4	5.5111	6.9956	26.9379	12.4505	13.8045	10.8751
5	6.1356	8.2553	34.5478	13.3844	15.3543	14.7181
6	6.8325	9.7534	42.7497	14.3829	17.0949	18.8558
7	7.6102	11.3727	49.4406	15.4502	18.7917	21.6279
8	8.4778	13.1416	55.0121	16.5906	20.8438	25.6357
9	9.4456	15.2574	61.5298	17.8090	22.9964	29.1282
10	10.5247	17.3844	65.1771	19.1103	25.2481	32.1174
11	11.7273	19.9983	70.5275	20.5002	27.7474	35.3519
12	13.0666	23.1748	77.3585	21.9847	30.5111	38.7835
13	14.5569	26.8443	84.4089	23.5705	33.4027	41.7137
14	16.2134	31.0127	91.2778	25.2650	36.8937	46.0266
15	18.0522	35.6022	97.2178	27.0765	40.4674	49.4558
16	20.0902	40.7135	102.6534	29.0141	44.7757	54.3243
17	22.3450	46.7144	109.0600	31.0880	49.3679	58.8006
18	24.8344	53.3229	114.7143	33.3096	54.9415	64.9419
19	27.5763	61.0529	121.3966	35.6920	60.9044	70.6391
20	30.5884	69.7168	127.9196	38.2492	67.8519	77.3942

IV. DISCUSSION OF RESULTS

For a random noise variation of 0.01 and 0.1 over repeated simulations as shown on Table 1 to table 4, we have observed a relatively smaller prediction of biodiversity gain whereas for a random noise variation of 0.8, we have observed a bigger prediction of biodiversity gain. On the basis of this present analysis, a random noise inclusion which may be considered as having a negative effect, has turned out in this scenario to have a positive effect on the ecological services.

V. CONCLUSION

Not all random noise driven factors do predict biodiversity loss. We have utilized the technique of a numerical simulation to predict that a higher random noise perturbation has the potential to predict bigger volumes of biodiversity gain than a lower random noise perturbation, provided the two yeast species interact mutually on the simplifying assumption of varying the inter-competition coefficients together. This numerical result complements a popular ecological idea that in a harsh ecological environment, species tend to benefit each other (Ekaka-a 2009, Ford, Lumb, Ekaka-a 2010). The predictions of this present study are based on the one hundred and fifty (150) percent variations of the inter competition coefficients together on the simplifying assumption that the intra-competition coefficients outweigh the inter-competition coefficients. However, we have not extended this idea to the scenario of two (2) competing yeast species undergoing a random noise perturbation. This will be the subject of our next investigation.

REFERENCES

[1] Atsu, J. U. & Ekaka-a, E. N. (2017). Modeling the policy implications of biodiversity loss: A case study

of the Cross River national park, south – south Nigeria. International Journal of Pure and Applied Science, Cambridge Research and Publications. vol 10 No. 1; pp 30-37.

[2] Atsu, J. U. & Ekaka-a, E. N. (2017). Quantifying the impact of changing intrinsic growth rate on the biodiversity of the forest resource biomass: implications for the Cross River State forest resource at the Cross River National Park, South – South, Nigeria: African Scholar Journal of Pure and Applied Science, 7(1); 117 – 130.

[3] De Mazancourt, C., Isbell, F., Larocque, A., Berendse, F., De Luca, E., Grace, J.B et al. (2013). Predicting ecosystem stability from community composition and biodiversity. Ecology Letters,, DOI: 10.1111/ele.12088.

[4] Ernest, S.K.M. & Brown, J.H. (2001). Homeostasis and compensation: the role of species and resources in ecosystem stability. Ecology, 82, 2118–2132.

[5] Fowler, M.S., Laakso, J., Kaitala, V., Ruokolainen, L. & Ranta, E. (2012). Species dynamics alter community diversity-biomass stability relationships. Ecol. Lett., 15, 1387–1396.

[6] Gonzalez, A. & Descamps-Julien, B. (2004). Population and community variability in randomly fluctuating environments. Oikos, 106, 105–116.

[7] Grman, E., Lau, J.A., Donald, R., Schoolmaster, J. & Gross, K.L. (2010). Mechanisms contributing to stability in ecosystem function depend on the environmental context. Ecol. Lett., 13, 1400–1410.

[8] Hector, A., Hautier, Y., Saner, P., Wacker, L., Bagchi, R., Joshi, J. et al. (2010). General stabilizing effects of plant diversity on grassland productivity through population asynchrony and overyielding. Ecology, 91, 2213–2220.

[9] Loreau, M.. & de Mazancourt, C.. (2013). Biodiversity and ecosystem stability: a synthesis of underlying mechanisms. Ecol. Lett., DOI: 10.1111/ele.12073.

[10] MacArthur, R. (1955). Fluctuations of Animal Populations, and a Measure of Community Stability. Ecology, 36, 533–536.

[11] Marquard, E., Weigelt, A., Roscher, C., Gubsch, M., Lipowsky, A. & Schmid, B. (2009). Positive biodiversity-productivity relationship due to increased plant density. J. Ecol., 97, 696–704.

[12] May, R.M. (1973). Stability and complexity in model ecosystems. 2001, Princeton Landmarks in Biology edn. Princeton University Press, Princeton. McCann, K.S. (2000). The diversity-stability debate. Nature, 405, 228–233.

[13] McNaughton, S.J. (1977). Diversity and stability of ecological communities: a comment on the role of empiricism in ecology. Am. Nat., 111, 515–525.

[14] Mutshinda, C.M., O'Hara, R.B. & Woiwod, I.P. (2009). What drives community dynamics? Proc. Biol. Sci., 276, 2923–2929.

[15] Proulx, R., Wirth, C., Voigt, W., Weigelt, A., Roscher, C., Attinger, S. et al.(2010). Diversity Promotes Temporal Stability across Levels of Ecosystem Organization in Experimental Grasslands. PLoS ONE, 5, e13382.