International Journal of Advanced Engineering, Management and Science (IJAEMS) *[Vol-3, Issue-2, Feb- 2017]*
https://dx.doi.org/10.24001/ijaems.3.2.6 *ISSN : 2454-1311*

Dynamic Instability of Viscoelastic Plate in Supersonic Flow

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Abstract— *The present work is investigating the aero-elastic instability of a viscoelastic plates under compressive forces. The Bubnov-Galerkin method used to solve the governing equations. The quasi-steady aerodynamic loadings are determined using linear piston theory. The nonlinear integro-differential equation of the plate is transformed into a set of nonlinear algebraic equations through a Galerkin approach. The resulting system of the equations is analytically solved. The influence of elastic and viscoelastic properties and the compressive load characteristicsof the plate material on the value of critical parameters are discussed.*

Keywords*— *Viscoelasticity, nonlinear panel flutter, viscoelastic plate; integro-differential equation, Bubnov-Galerkin method.

Nomenclature

D	= bending rigidity of the panel
m	=mass unit of the panel
ΔP	=aerodynamic loading
N_x	=external compressive load
R^*	=relaxation operator
$R(t-\tau)$	=relaxation kernel
t	=time
h	= thickness of the panel
γ	=polytrophic exponent
ρ_0	=air density
c_0	=speed of sound
$\Gamma[\alpha]$	= Euler gamma-function

I. INTRODUCTION

The application of viscoelastic materials in structural elements like plates or shells during last decades is rapidly grown. The use of viscoelastic material as vibration induced elements and dampers used in structures in particular the flying vehicle structure has grown. Consequently, the analyses of this type of structures have gained momentum during recent years. There are many works published which deal with different aspects of viscoelastic behavior of composite plates [1-4]. Some of them are reviewed below.

Destabilization effect of the internal friction in a material was mentioned first in [5] and later in other works [6-7]. The buckling analysis of viscoelastic plates subjected to dynamic loading in a nonlinear formulation with weak singular relaxation kernel is presented in [8-10]. The dynamic stability of fiber-reinforced laminated rectangular plates used first-order shear deformation theory was carried out in [11]. Viscoelastic body model is utilized to describe material damping was discussed in [12]. This relates the stability problem for elastic systems with that for viscoelastic systems [10, 12]. The Bubnov-Galerkin method is usually applied for solving the problems. It is important to study the instability of viscoelastic system with the lateral compressive force being taken into account. In the present, work the problem of the instabilityis solved for a viscoelastic plate in a supersonic gas flow. The influence of viscoelastic properties of the plate material, the external and aerodynamic damping, the flow speed, and compressive load characteristic on the value of critical parameters is discussed.

II. THE STATEMENT OF THE PROBLEM

2.1. A long viscoelastic plate

This work considersaninfinitely long viscoelastic plate with freely supported longer edges exposed to a supersonic flow with a constant velocity V (Fig.1).

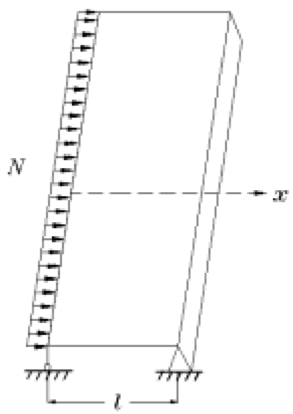

Fig.1: The plate in a gas flow

https://dx.doi.org/10.24001/ijaems.3.2.6 **ISSN : 2454-1311**

The plate is subjected to a compressive load in the plane of the plate. The load $q(t)$, which is uniformly distributed along the edge, is applied in the neutral plane of the plate. Because of thearbitrary local curvature,the surface of a plate is deformed. It is assumed that the plate deflection W is a function of the single space coordinate x and time t, i.e. $W = W(x,t)$. This shape of the deflection entails occurrences of changes the distribution of the aerodynamic forces. The aerodynamic forces are subject tothe fluctuations of the viscoelastic plate. These fluctuations have a nonsymmetrical form and damped,because of the hereditary deformation of the material of the plate. These deformations are described by relation

$$\sigma = E[\varepsilon(x,t) - R^*\varepsilon(x,t)], \quad (1)$$

where

$$R^*\varepsilon = \int_{-\infty}^{t} R(t-\tau)\varepsilon(x,\tau)d\tau,$$

$$R(t-\tau) = \varepsilon e^{-\beta(t-\tau)}(t-\tau)^{\alpha}, \ \varepsilon > 0, \ \beta > 0, \ 0 < \alpha < 1 \quad (2)$$

$$0 \le \int_0^{\infty} R(\tau)d\tau < 1$$

Using the linear piston theory, the equation of motion of the plate is written in the following form:

$$D(1-R^*)\frac{\partial^4 W}{\partial x^4} + N_x\frac{\partial^2 W}{\partial x^2} + m\frac{\partial^2 W}{\partial t^2} + \Delta P = 0, \quad (3)$$

where $D = \dfrac{Eh^3}{12(1-\mu^2)}$ is the stiffness of the plate, E, μ are the elasticity modulus and the Poisson ratio of the plate material, h is the plate thickness. The term ΔP determine the aerodynamic load, defined by the piston theory

$$\Delta P = \frac{\gamma P_0}{c_0}\left(\frac{\partial W}{\partial t} + V\frac{\partial W}{\partial x}\right), \quad (4)$$

and N_x is the intensity of forces, acting the neutral surface of the plate.

The solution of the integro-differential equation(IDE) (3) must satisfy the boundary conditions: at $x=0$ and $x=l$, and initial conditions at $t=0$

$$W(0)=0, \quad \frac{\partial^2 W}{\partial x^2}=0 \quad (5)$$

Now by introducingthe dimensionless coordinate $\frac{x}{l}$ and speed $M = \frac{V}{C_0}$, then IDE (3)can berewritten in the dimensionless form

$$(1-R^*)\frac{\partial^4 W}{\partial x^4} + \overline{N}\frac{\partial^2 W}{\partial x^2} + a_3 M\frac{\partial W}{\partial x} + a_1\frac{\partial^2 W}{\partial t^2} + a_3\frac{\partial W}{\partial t} = 0 \quad (6),$$

where $\overline{N} = \dfrac{Nl^2}{D}; \ a_1 = \dfrac{ml^4}{D}; \ a_2 = \dfrac{\gamma P_0 l^4}{C_0 D}; \ a_3 = \dfrac{\gamma P_0 l^3}{D}$

An approximate solution of equation (3) is sought in the form

$$W(x,t) = \sum_{n=1}^{m} f_n(t)\sin n\pi x, \quad (7)$$

where $f_n(t)$ is unknown function of time. To obtain the function $f_n(t)$ the Bubnov-Galerkin method will be used. Substituting expression (7) intoEq. (6), multiplying by $\sin n\pi x$ and integrating with respect to x on the interval $[0,l]$, gives as the final expression

$$\frac{\partial^2 f_n}{\partial t^2} + c\frac{\partial f_n}{\partial t} + \omega_n^2\left[(1-R^*) - \frac{r}{n^2}\right]f_n + PM\sum_{j=1}^{m} b_{nj} f_j = 0, \quad (8)$$

where

$$c = \frac{a_2}{a_1}, \ \omega_n^2 = \frac{1}{\alpha_1}(n\pi)^4, \ r = \frac{N}{\pi^2 D}, \ P = \frac{\gamma P_0 l^3}{m}.$$

Here the term

$$b_{nj} = 2\pi j\int_0^1 \sin\pi n x\cos\pi j x dx = 2j\int_0^{\pi}\sin nz\cos jz dz = \begin{cases} 4\dfrac{nj}{n^2-j^2} & \text{if } (n \ne j) \\ 0 & \text{if } (n=j) \end{cases}.$$

Now if the solution of IDE (8) is searched in theform of expression

$$f_n(t) = A_n\sin\omega t, \quad (9)$$

then has been thenext integral identity

$$A_n(t-R^*)\sin\omega t = A_n(1-R_c+R_s) \quad (10)$$

easilyapplied, where

$$R_c = \int_0^{\infty} R(\tau)\cos\omega\tau\, d\tau = \varepsilon\frac{\Gamma[\alpha]\cos\alpha\theta}{[\beta^2-\omega^2]^{\frac{\alpha}{2}}}, \quad \theta = -arctg\frac{\omega}{\beta}$$

$$R_s = \int_0^{\infty} R(\tau)\sin\omega\tau\, d\tau = \varepsilon\frac{\Gamma[\alpha]\sin\alpha\theta}{[\beta^2-\omega^2]^{\frac{\alpha}{2}}}.$$

From the equation (8) at $m=2$, onecan obtain the expressions

$$c\omega + \omega_n^2 R_s = 0 \quad (11)$$

$$\left.\begin{aligned}&\left\{\omega_1^2\left[(1-R_c)-r\right]-\omega^2\right\}A_1-\frac{8}{3}PMA_2=0\\&\frac{8}{3}PMA_1+\left\{\omega_2^2\left[(1-R_c)\right]-\omega^2\right\}A_2=0\end{aligned}\right\}\quad ,(12)$$

where

$$\omega_1^2=\frac{\pi^4}{a_1},\quad \omega_2^2=\frac{16\pi^4}{a_1}.$$

The system of the equations(12) can be rewrittenas

$$\left.\begin{aligned}&\left[(1-R_c)-r-\varpi^2\right]A_1-\frac{8}{3}PMA_2=0\\&\frac{8}{3}qA_1+\left[16(1-R_c)-4r-\varpi^2\right]A_2=0\end{aligned}\right\},\quad (13)$$

where $\varpi=\frac{\omega}{\omega_1},\quad q=\frac{PM}{\omega_1}$.

When the determinant of the Eq. (13) is equal to zero, one can obtain the reduced velocity of the flow as

$$q=\frac{3}{8}\sqrt{\left[\varpi^2+r-(1-R_c)\right]\left[16(1-R_c)-4r-\varpi^2\right]},\quad (14)$$

where r is compressive load on the plate. Eq. (14) is represent relation of the reduced velocity of flow q with the frequency ϖ .Let demonstrate this relation in the examples investigated below, where r has different value.

First,it is assumed $r=0$, then Eq. (14) can be rewritten as following expression

$$q=\frac{3}{8}\sqrt{\left[\varpi^2-(1+R_c)\right]\left[16(1-R_c)-\varpi^2\right]}.\quad (15)$$

From conditions respectively to $\frac{dq}{d\omega}=0$ for the equations (14) and (15), onecan find

$$\varpi^2=\frac{17}{2}(1-R_c)-\frac{5}{2}r,\quad (16)$$

$$\varpi^2=\frac{17}{2}(1-R_c).\quad (17)$$

In case of the perfect elastic plate, whereparameters are $\varepsilon=0,\quad R_c=0$ the equations (16) and (17) willbe written in the following form, respectively

$$\varpi^2=\frac{17}{2}-\frac{5}{2}r,\quad (18)$$

$$\varpi^2=\frac{17}{2}\quad (18a)$$

The maximum value of the reduced velocity q_f will be

$$q_f^{(v)}=\frac{45}{16}(1-R_c)-\frac{3}{2}r;\quad q_f^{(e)}=\frac{45}{16}-\frac{3}{2}r\quad (19)$$

$$q_f^{(v)}=\frac{45}{16}(1-R_c);\quad q_f^{(e)}=\frac{45}{16}=2.81.\quad (20)$$

If substitute $R_0=1-R_c$ into the equations (19),and (20), then can be obtained next expressions

$$q_f^{(v)}=q_f^{(e)}R_0-\frac{3}{2}rR_c,\quad (21)$$

$$q_f^{(v)}=q_f^{(e)}(1-R_c)=q_f^{(e)}R_0.\quad (22)$$

New inequality $0<1-R_c=R_0<1$ based onthe inequalities (2) and $0<R_c<1$ was introduced. Under this inequality the Eq. (21) and (22) are analyzed for relaxation kernel $R_0=0.5$. This analyze demonstratesdecreasing of the critical values of the reduced velocity for the viscoelastic case $q_f^{(v)}$ twice then reduced velocity in the elastic case $q_f^{(e)}$. When we substitute relaxation kernel $R_0=0.4$ into Eq. (21) and (22) the critical valueof the reduced velocity for the viscoelastic case $q_f^{(v)}$ is decreasing and it is two-and-a-half time less then $q_f^{(e)}$. Such reduction of the reduced velocity $q_f^{(v)}$ is observedonly because of the hereditary-deformable properties of the material of the plate, such as the viscosity- ε, the relaxation- β, and the singularity- α. Consequently, it will affect to the corresponding value of the critical speed of flutter- $V_f^{(v)}$.

The same effect was explored at weak singularity kernels of the hereditary the transient process of the panel flutter in [10]. The functional dependence of q on ϖ^2 is illustrated in Figure 2.

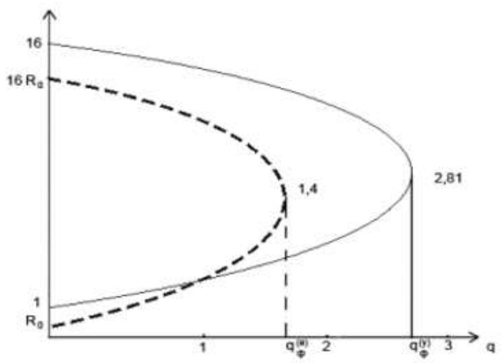

Fig.2: The calculation of reduced velocity

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Let now investigate the second case, when compressive load $r \neq 0$.

The equations (11) and (12) wereobtained, when the conditions $\cos wt = 0$ and $\sin wt = 0$ respectively. The analogously to them can be obtained the solution of IDE (8) in the form of $f_n(t) = A_n e^{i\omega t}$. Here A_n is the constant, ω is the frequency and $i = \sqrt{-1}$.

Then this will be the complex expressions in the following form

$$\Delta = \Delta_1 + i\Delta_2 = 0; \quad \Delta_1 = 0, \Delta_2 = 0.$$

Here are

$$\Delta_1 = c\omega + \omega_n^2 R_S = 0;$$

$$\Delta_2 = \begin{cases} \left[\omega_1^2(1-R_c) - r - \omega^2\right]A_1 - \frac{8}{3}PMA_2 = 0 \\ \frac{8}{3}PMA_1 + \left[\omega_2^2(1-R_c) - \frac{r}{4} - \omega^2\right]A_2 = 0 \end{cases} \quad (23a)$$

From the condition of the existence of the nontrivial solutions (23a),we can obtain the expressions similar to the expressions (14) and (15).

Thus, both approaches of the solution of IDE (8) give identical results. It is significant, when the loads N are compressive as can be seen from the equation (21), then it leads to a decrease V_{cr}, and when N are the stretching forces, then it leads to an increase in the critical speed of flutter.

2.2.The rectangular viscoelastic plate

Next,consider the case of the rectangular plate. This problem has the same governing equation and boundary conditions as in 2.1.Assuming that the plate is compressed along the x axis by forces N ,and the equation (6) can be rewritten inthe following form

$$(1 - R^*)\Delta^2 W + \overline{N}\frac{\partial^2 W}{\partial x^2} + a_3 M\frac{\partial W}{\partial x} + a_1\frac{\partial^2 W}{\partial t^2} + a_2\frac{\partial W}{\partial t} = 0, \quad (23)$$

where

$$\overline{N} = \frac{Na^2}{D}; \quad a_1 = \frac{ma^4}{D}, \quad a_2 = \frac{\gamma P_0 a^4}{c_0 D}, \quad a_3 = \frac{\gamma P_0 a^3}{D}, \Delta^2 W = W_{xxxx} + 2\lambda W_{xxyy} + a^4 W_{yy}, \quad \lambda = \frac{a}{b}.$$

Assuming, that $W = \overline{W}\sin\omega t$, the equation relative to $\overline{W} = \overline{W}(x,y)$ becomes

$$X = \left[(1-R_c)\Delta^2\overline{W} + N\overline{W}_{xx} + a_3 M\overline{W}_x - a_1\omega^2 W\right]\sin\omega t + \left[a_2\omega\overline{W} + R_S\Delta^2 W\right]\cos\omega t = 0 \quad (24)$$

By using the approximation expression for a deflection

$$\overline{W}(x,y) = (A_1\sin\pi x + A_2\sin 2\pi x)\sin\pi y, \quad (25)$$

one extract Bubnov-Galerkin equation for the rectangular platewith $\lambda = 1$. This leads to the equations

$$c\overline{\omega} + \overline{\omega}_n^2 R_s = 0, \quad n = 1,2 \quad (26)$$

$$\left.\begin{aligned} &\{[1(-R_c) - r] - \omega^2\}A_1 - \frac{2}{3}qA_2 = 0 \\ &\frac{2}{3}qA_1 + \left[\frac{25}{4}(1-R_c) - 4r - \overline{\omega}^2\right]A_2 = 0 \end{aligned}\right\}, \quad (27)$$

where

$$\overline{\omega} = \frac{\omega}{\omega_1}, \omega_1 = 2\pi^2\sqrt{\frac{D}{m}}, \quad \omega_2 = 5\pi^2\sqrt{\frac{D}{m}}.$$

The determinant of the system (27) is equal to zero and itallows to findreduced velocity

$$q = \frac{3}{2}\sqrt{\left[\overline{\omega}^2 + r - (1-R_c)\right]\left[\frac{25}{4}(1-R_c) - 4r - \overline{\omega}^2\right]}. \quad (28)$$

From the condition $\frac{\partial q}{\partial \overline{\omega}} = 0$, it can be found that

$$\overline{\omega}^2 = \frac{29}{8}(1-R_c) - \frac{5}{2}r. \quad (29)$$

The maximum value of q, which is consider as criticalis equal to

$$q_f^{(v)} = \frac{21}{8}(1-R_c) - \frac{3}{2}r = \frac{3}{2}\left[\frac{7}{4}(1-R_c) - r\right] \quad (30)$$

In aperfect elastic case, when $\varepsilon = 0, \quad R_c = 0$ from the expression (30) follows that

$$q_f^{(e)} = \frac{3}{2}\left[\frac{7}{4} - r\right]. \quad (31)$$

By substituting equation (31) into the expression (30) is obtained

$$q_f^{(v)} = q_f^{(e)}(1-R_c) - \frac{3}{2}rR_c. \quad (32)$$

In the absence of the compressing loads $(r = 0)$ it will be

$$q_f^{(e)} = \frac{21}{8}; \quad q_\infty^{(v)} = q_\infty^{(e)} R_0, \quad R_0 = 1 - R_c \qquad (33)$$

In the Figure 3 shown the formation of the loop of static equilibrium and straight lines (31) and (32) are the points of contact of tangency that correspond to the values q and r of the perfect elastic and hereditary- deformed rectangular plate.

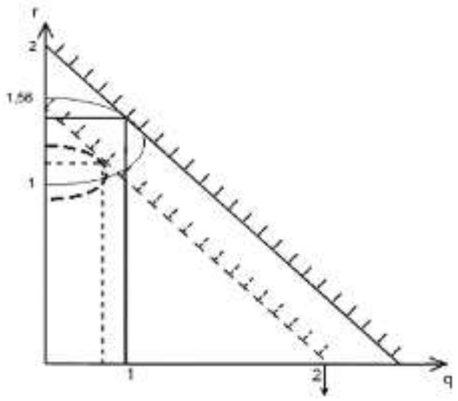

Fig.3: The calculation of q for the perfect elastic and viscoelastic rectangular plates

III. CONCLUSION

In this work, the effect of compressive load to the stability of the elastic and viscoelastic plate in the gas flow has been investigated. It has been observed, when shear load is indicated factor, it can have influence on the stability of the viscoelastic systems. The values of critical parameters for the elongated and the rectangular plate are strong evidence of no validity of the conclusions given in [11-13] concerning hereditary deformable properties of the materials. First, because of the integral operator is directly depend on the relaxation parameters ε, β, and α. Second, both of the values of the reduced velocities $q_f^{(v)}$ and $q_f^{(e)}$ have not identical representation, and essentially differ from each other.The examples with relaxation kernels clearly demonstrate reduced velocity differences between perfect elastic and viscoelastic cases. Thus, when relaxation kernels are $R_0 = 0.5$, and $R_0 = 0.4$ were observed reduced velocity difference between perfect elastic and viscoelastic plate 2 and 2.5 times. Hence, the application of the hereditary deformable (viscoelastic) plate in the transient and the steady process leads to an essential reduction of the critical speed of flutter.

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