Eigen Subspace based Direction of Arrival Estimation for Coherent Sources

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Abstract— *Direction of arrival (DOA) estimation technology plays an important role in enhancing the performance of the adaptive arrays for mobile communication. In this paper comparative performance analysis of eigen subspace based DOA estimation for coherent sources is presented. A number of DOA estimation algorithms based on eigen subspace method have been developed. Among these MUSIC algorithm is considered to have exceptionally good results. The focus of this paper is to unveil the performance characteristics of MUSIC algorithm and its improved version for coherent sources. The simulation results show that the improved MUSIC algorithm is the best. Also it can be observed that the resolution of DOA estimation improves as the number of snapshots and signal to noise ratio increases.*

Keywords— Coherent sources, Directional of arrival (DOA), eigen subspace, MUSIC.

I. INTRODUCTION

The demand for faster and more efficient telecommunication system has been skyrocketing, which has created a huge demand for spectral efficiency. The capacity can be improved either by enlarging its frequency bandwidth or allocating new spectrum, but these are impractical and expensive. Instead of finding more spectrum for serving same number of users different transmission techniques are adapted so that system can serve more users with same amount of spectrum. These transmission techniques include Frequency Division Multiplexing (FDMA), Time Division Multiplexing (TDMA), Code Division Multiplexing (CDMA) and Space Division Multiplexing (SDMA). The deployment of smart antenna system (SAS) comes into picture during employment of SDMA for wireless communication. Some of the potential benefits of SAS include increased frequency reuse, null steering, multipath mitigation, improved array resolution, instantaneous tracking of moving sources etc. The performance of SAS greatly depends on the performance of DOA estimation [1]..

II. DOA ESTIMATION ALGORITHMS

The DOA estimation algorithms are classified as quadratic type and eigen subspace type. The quadratic method is highly dependent on physical size of array aperture which results in poor accuracy and resolution [6]. Eigen subspace based DOA estimation utilizes eigen decomposition. One such method discussed is MUSIC and improved MUSIC algorithm.

2.1 MUSIC ALGORITHM

MUSIC stands for MUltiple SIgnal Classification [4][5]. It is characterized by the eigen decomposition of covariance matrix (Rx). The covariance matrix is given by,

$$Rx= ARsA^{H}+ \sigma^{2}I \quad (1)$$

Where, $A= a_m(\theta_k)=\exp[\frac{(-j(m-1)\,2d\pi\sin\theta_k)}{\lambda}]$ (2)

m = (1,2,….,M) and k = (1,2,….,D)
A is a MxD array steering matrix.
Rs is a DxD signal correlation matrix
σ^2 is the noise variance and I is the MxM identity matrix.
In which M is the number of array elements with identical response and D is the number of coherent signal sources. The eigen values and eigen vectors which belong to Rx are corresponding to signal and noise respectively. Therefore, the eigen value (eigenvector) of Rx can be divided as signal eigen value (eigenvector) and noise eigen value (eigenvector). The covariance matrix written in terms of eigen values and eigen vectors as,

$$Rx=\sum_{i=1}^{M} \lambda_i v_i v_i^H \quad (3)$$

Where λ is the eigen value and v is the corresponding eigen vector. If λ_i be the i-th eigen values of the matrix Rx, v_i is eigenvector corresponding to λ_i, then,

$$Rxv_i= \lambda_i v_i, \quad \text{where } (i= 1, 2,\dots,M) \quad (4)$$

For the analysis purpose the minimum eigen value of Rx is considered as σ^2

$$Rxv_i=\sigma^2 v_i \quad \text{where } (i=D+1,D+2,\dots,M) \quad (5)$$

From the (1) we get $\sigma^2 v_i = (ARsA^H + \sigma^2 I)\, v_i$, (6)

$$ARsA^H\, v_i = 0 \tag{7}$$

Since $A^H A$ is a DxD full rank matrix. Therefore $(A^H A)^{-1}$ exits and Rs^{-1} also exits. So we get,

$A^H\, v_i = 0$ where (i=D+1, D+2,…,M). (8)

The above equation indicates that the eigen vector corresponding to noise eigen value v_i is orthogonal with the column vector of the matrix A. Using noise characteristics value as each column noise matrix E_n can be constructed in terms of eigen vector as,

$E_n = [v_{D+1}, v_{D+2}, \ldots\ldots\ldots\ldots, v_M]$ (9)

This orthogonality principle shows that the Euclidean distance $d^2 = a(\theta)^H E_n E_n^H a(\theta) = 0$ to each and every angle $(\theta_1, \theta_2, \ldots .\theta_D)$. This distance expression in the denominator creates sharp peaks at directional of arrival.

Therefore the spatial spectrum of MUSIC algorithm $P_{mu}(\theta)$ can be defined as

$$P_{mu}(\theta) = \frac{1}{a(\theta)^H E_n E_n^H a(\theta)} = \frac{1}{\left\| E_n^H a(\theta) \right\|^2} \tag{10}$$

Where, $a(\theta)$= steering vector ,

E_n = noise matrix.

By this formula estimate of the arrival angles is found by finding the peak.

2.2 IMPROVED MUSIC ALGORITHM

MUSIC algorithm can theoretically achieve an arbitrarily high resolution to estimate DOA. However, for MUSIC algorithm is limited to uncorrelated signals. When the source is a correlated signal or a signal with low SNR the estimated performance of the MUSIC algorithm deteriorates. In order to combat this, improved MUSIC algorithm is implemented, which employs conjugate reconstruction of the data matrix of the MUSIC algorithm [7]. M^{th}-order transformation matrix J is used.

$$J = \begin{bmatrix} 0 & \cdots & 1 \\ \cdots & \cdots & \cdots \\ 1 & \cdots & 0 \end{bmatrix} \tag{11}$$

Using J , covariance of data matrix is constructed as

$$R = ARsA^H + J[ARsA^H]^* J + 2\sigma^2 I \tag{12}$$

On performing characteristic decomposition of R and its eigen value and eigen vector are obtained. According to the estimated number of signal source, noise subspace is separated, and then spatial spectrum is constructed to obtain the estimated DOA value by finding the peak.

Therefore the spatial spectrum of Improved MUSIC algorithm $P_{im}(\theta)$ is coined using the covariance matrix given by equation (12) can be defined as,

$$P_{im}(\theta) = \frac{1}{a(\theta)^H E_n E_n^H a(\theta)} = \frac{1}{\left\| E_n^H a(\theta) \right\|^2} \tag{13}$$

III. SIMULATION RESULTS

MUSIC and improved MUSIC algorithms are simulated using Matlab. The performance is analyzed by considering coherent signals, with an incident angle of 20^0 and 30^0 respectively, with ideal Gaussian white noise, SNR of 20dB, element spacing of half the input signal wavelength, array element size of 10 and 200 snapshots.

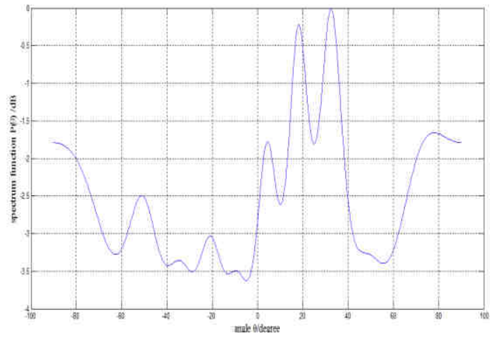

Fig.1: Power spectrum plot (power(dB) v/s angle (degrees)) of MUSIC algorithm.

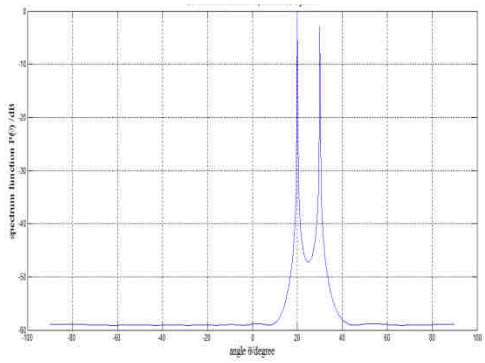

Fig.2: Power spectrum plot (power (dB) v/s angle (degrees)) of improved MUSIC algorithm.

From the simulations (Figure 1and 2) it is evident that classic MUSIC algorithm has lost effectiveness, while improved MUSIC algorithm can be better applied to remove the signal correlation feature, which can distinguish the coherent signals, and estimate the angle of arrival more accurately.

On comparing the MUSIC and improved MUSIC for varying value of SNR, Figure 3and 4 indicates that, as the SNR value decreases peaks in the spectrum start to disappear ie. reduction in the resolution capability.

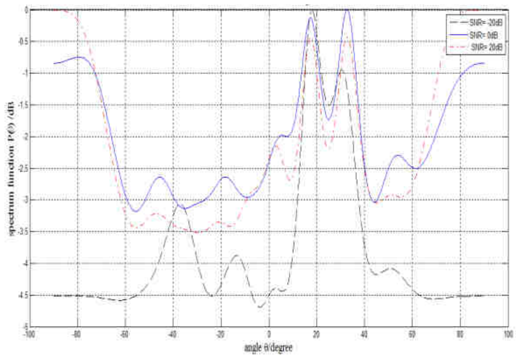

Fig.3: Power spectrum of MUSIC for varying value of SNR.

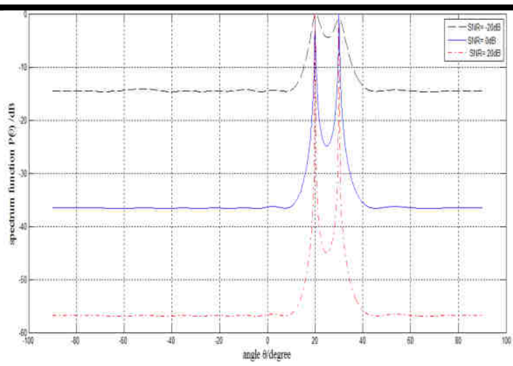

Fig.4: Power spectrum of improved MUSIC for varying value of SNR.

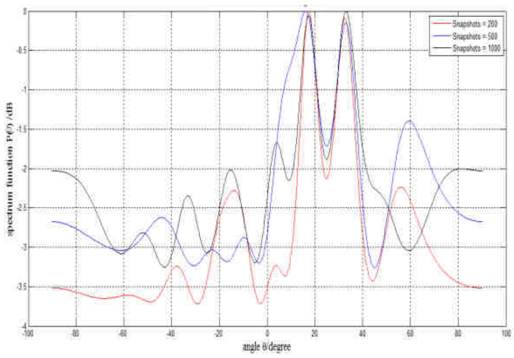

Fig.5: Power spectrum of MUSIC for varying snapshots.

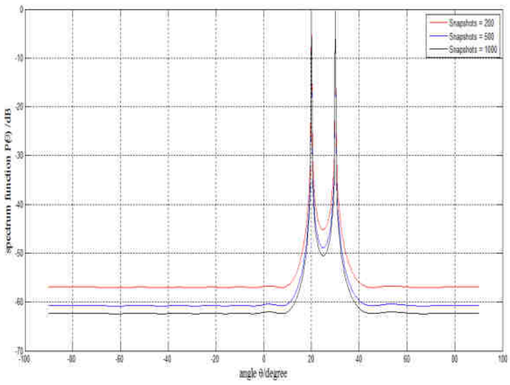

Fig. 6: Power spectrum of improved MUSIC for varying snapshots.

Figure 5and 6 indicates MUSIC and improved MUSIC spectrums for varying number of snapshots that increase from 200 to 1000. It can be seen that the resolution capability increases with snapshots and the signals can be clearly recognized

On comparing these algorithms for varying SNR value (snr= -20dB, 0dB, 20dB) with snapshots of 200 and varying number of snapshots (N=200, 500,1000) with SNR of 20 dB , the results obtained are tabulated in Table(1 and 2).

Table 1: DOA estimation by (i) MUSIC and (ii) improved MUSIC for varying SNR.

DOA	snr=-20dB		snr=0dB		snr=20dB	
	(i)	(ii)	(i)	(ii)	(i)	(ii)
20^0	17^0	20.5^0	18.5^0	20^0	18^0	20^0
30^0	32.5^0	29.9^0	31^0	30^0	33^0	30^0

Table 2: DOA estimation by (i) MUSIC and (ii) improved MUSIC for varying snapshots.

DOA	N=200		N=500		N=1000	
	(i)	(ii)	(i)	(ii)	(i)	(ii)
20^0	16^0	20^0	17.5^0	20^0	18^0	20^0
30^0	33^0	30^0	33^0	30^0	32.5^0	30^0

From the results obtained it is evident that the improved MUSIC outperforms the MUSIC algorithm at scenarios such as varying SNR and snapshots for coherent signals.

IV. CONCLUSION

This paper presents the DOA estimation for coherent signals based on eigen subspace method, where in the two methods MUSIC and improved MUSIC are investigated. MUSIC though is an effective method to distinguish between arrival angles, fails for coherent sources. If the sources are not coherent the data covariance matrix is strictly a diagonal matrix. But for coherent sources they are not due to the divergence of signal eigen vectors into the noise subspace hence reducing the accuracy. As a means to combat this, improved MUSIC can be implemented. From the results obtained it can be concluded that the improved variant has better performance capability comparatively. The results showcase that the algorithm improves with increase in SNR and number of snapshots. The improvements are analysed in the form of sharper peaks. Therefore this study upholds a new potential possibility of effective user separation through SDMA that can be implemented for mobile communication [2][3][8].

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