A Novel Algorithm for Cooperative Spectrum Sensing in Cognitive Radio Networks

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Abstract—The rapid growth in wireless communication technology has led to a scarcity of spectrum. But, studies are saying that licensed spectrum is underutilized. Cognitive Radio Networks (CRNs) seem to be a promising solution to this problem by allowing unlicensed users to access the unused spectrum opportunistically. In this paper we proposed a novel spectrum sensing algorithm to improve the probabilities of detection and false alarm in a CRN, using the traditional techniques of energy and first order correlation detection. Results show a significant improvement in performance in cooperative spectrum sensing.

Keywords—cognitive radio networks, energy detection, correlation detection, cooperative spectrum sensing.

I. INTRODUCTION

CRNs have been emerging as a promising solution for the shortage of spectrum in wireless communication. The design of any CRN is based on opportunistic usage of licensed spectrum without causing interference to the primary user (PU) or licensed user. Spectrum sensing is the first step of the working of a cognitive radio network. The others being analysis, decision and adapting.

Spectrum sensing is broadly classified as three types; non-cooperative, cooperative and interference based. We use the non-cooperative techniques of energy and correlation detection techniques to propose an algorithm for cooperative spectrum sensing. Both energy and correlation detection uses a threshold to make a decision on whether the spectrum is occupied by PU or free for the use of secondary user (SU) or cognitive radio user.

A traditional energy detection is proposed in [2] and the assumption in the technique is that the energy of the signal is higher than the threshold, if PU is present. This information is used to make a decision by the SU. Authors in [5] used covariance information of received samples to determine the presence or absence of PU. The threshold is generated by using a covariance matrix of received samples by assuming the desired sample gives a higher covariance if PU is present. This threshold is used to make a decision for the future received signals.

In this paper we propose a solution for cooperative spectrum sensing using correlation based detection. Statistics we employed to measure the improvement are probability of detection \((P_d)\) and probability of false alarm \((P_f)\). Probability of detection being the chance of SU detecting the presence or absence of PU accurately and probability of false alarm being the system showing that a PU is present even if the spectrum is free, rendering the SU losing the opportunity to use the spectrum. We first give a novel summary statistic as the first order correlation of the received samples. Sensing is formulated based on energy and correlation detection. We derive the probabilities of detection and false alarm for different scenarios of spectrum sensing using linear combination of the summary statistics. The linear combination gives a lower computational complexity and closed form expressions. Also it can lead to a new statistic where the distance between the mean of the distributions for both hypotheses tests is higher. This ensues in a lower variance for any given SNR. The assumption in this scheme is that the energy and first order correlation of the samples is known. Because of this information, if the signal samples are correlated, the proposed algorithm significantly improves the performance of spectrum sensing. Nonetheless, if the signals are not correlated, then this algorithm acts as a traditional energy detection scheme as the correlation part of the summation becomes zero. The simulation results conveys that the proposed cooperative spectrum sensing algorithm improves both the probability of detection and probability of false alarm of a CRN.

The rest of the paper is structured as follows. Section II we define the system model with energy and first order correlation summary statistics. Section III deals with the local and global scenarios for spectrum sensing and derivations of probabilities of detection and false alarm in those scenarios. In Section IV, numerical outcomes are represented. Conclusion is give in Section V and last section enlists the references made to create this paper.

II. SYSTEM MODEL

We considered a CRN with M users. \(H_0\)represents the absence of a PU and \(H_1\)represents the presence of a PU. With these two hypotheses at \(k\)th time instance where, \(k=1,2,....,N\), the received signal \(x_i(t)\) by the i-th SU is given by
\( H_0: x_i(t) = v_i(t), \)
\( H_1: x_i(t) = v_i(t) + h_i s(k), i = 1, 2, 3, ..., M, \)
(1)

where \( v_i(t) \) and \( s(k) \) denote the additive white Gaussian noise (AWGN) for the \( i \)-th SU and the transmitted signal of PU at time \( k \). \( h_i \) denote the channel gain between the primary transmitter and the SU \( i \) receiver. We assume that the exact channel power gains can be estimated if the SUs know the primary transmit power. We assume that the transmitted signal \( s(k) \) and the set of noises \( \{v_i(k)\} \) are independent of each other. Usually the samples of transmitted signals are related to the modulation schemes employed, where adjacent samples of the transmitted pulses are correlated with each other. Thus, the sampling rate is chosen in a way that the set of noise samples are independent of each other.

SUs calculate two statistics \( u_i \) and \( r_i \) which denotes received energy and first order correlation of SU \( i \) over an interval of \( N \) samples, as follows

\[
u_i = \sum_{k=0}^{N-1} |x_i(k)|^2, \quad i = 1, 2, 3, ..., M,\]
(2)

\[
r_i = \sum_{k=0}^{N-2} x_i(k+1)x_i^*(k), \quad i = 1, 2, 3, ..., M,\]
(3)

Assuming \( |x_i(k)|^2 \) for \( i = 1, 2, ..., M \) to be independent and identically distributed (IID) random variables, and based on the central limit theorem [6], for large values of \( N \), the statistics \( \{u_i\} \) are approximately normally distributed with

\[
E[u_i] = \begin{cases} 
N\sigma_i^2, & H_0 \\
(N + \zeta_i)\sigma_i^2, & H_1
\end{cases},
\]
(4)

where \( \zeta_i = \frac{E[|h_i|^2]}{\sigma_i^2} \)

We define \( E_s \) as

\[
E_s = \sum_{k=0}^{N-1} |s(k)|^2.
\]
(5)

Variance is given by

\[
\text{Var}[u_i] = \begin{cases} 
N\sigma_i^4, & H_0 \\
(N + 2\zeta_i)\sigma_i^4, & H_1
\end{cases}.
\]
(6)

The same can be applied to \( r_i \) with a slight change of the terms \( x_i(k+1)x_i^*(k) \) and \( x_i(k+2)x_i^*(k+2) \) being not independent of each other. So we consider them as two IID random variable sets. By central limit theorem [6], for adequately higher values of \( N \), each of these sets approximately follows a normal distribution. As a result, statistics \( \{r_i\} \) have a near normal distribution with means

\[
E[r_i] = \begin{cases} 
0, & H_0 \\
|s_i|^2 R_s(1), & H_1
\end{cases},
\]
(7)

where

\[
R_s(1) = \sum_{k=0}^{N-2} s_i(k+1)s_i^*(k),
\]
(8)

and variance is

\[
\text{Var}[r_i] = \begin{cases} 
(N - 1)\sigma_i^4, & H_0 \\
(N - 1)\sigma_i^4 + 2E_s|s_i|^2\sigma_i^2, & H_1
\end{cases}.
\]
(9)

In the above equation we make an assumption that for large values of \( N, \sum_{k=0}^{N-2} |s(k+1)|^2 + |s(k)|^2 = 2E_s \).

The covariance of \( u_i \) and \( r_i \) is

\[
\text{Cov}[u_i, r_i] = \begin{cases} 
0, & H_0 \\
2|s_i|^2\sigma_i^2 R_s(1), & H_1
\end{cases}.
\]
(10)

![Block Diagram of System Model](https://example.com/block-diagram.png)

### III. LOCAL AND GLOBAL SCENARIOS OF SPECTRUM SENSING

(a) LOCAL SENSING

In local spectrum sensing, we examine each SU individually. Each SU \( i \) employs a linear summation of \( u_i \) and \( r_i \) as a test statistic. It is defined as

\[
z_i = w_{u,i} u_i + w_{r,i} r_i, \quad i \in \{1, 2, ..., N\}
\]

wherew\( w_{u,i} \) and \( w_{r,i} \) denote the combining weights for statistics \( u_i \) and \( r_i \), respectively. These coefficients signify the contribution of each statistic to the decision. Based on the above equation (11) and, equations (4) and (7), the mean of \( z_i \) is

\[
E[z_i] = \begin{cases} 
w_{u,i}N\sigma_i^2, & H_0 \\
w_{u,i}(N + \zeta_i)\sigma_i^2 + w_{r,i}|s_i|^2\sigma_i^2 R_s(1), & H_1
\end{cases}.
\]
(12)

By (6), (9) and (10), the variance of \( z_i \), is given by

\[
\text{Var}[z_i] = \begin{cases} 
w_{u,i}^2\sigma_i^4, & H_0 \\
w_{u,i}(N + \zeta_i)\sigma_i^4 + w_{r,i}|s_i|^2\sigma_i^4 R_s(1), & H_1
\end{cases}.
\]
(12)

wherew\( w_{u,i} = \{w_{u,i}, w_{r,i}\} \) and

\[
\Sigma_{H_0} = \begin{bmatrix} N\sigma_i^4 & 0 \\ 0 & (N+1)\sigma_i^4 \end{bmatrix},
\]
(14)

and

\[
\Sigma_{H_1} = \begin{bmatrix} (N + 2\zeta_i)\sigma_i^4 & 2|s_i|^2\sigma_i^4 R_s(1) \\ 2|s_i|^2\sigma_i^4 R_s(1) & 2E_s|s_i|^2\sigma_i^4 + (N-1)\sigma_i^4 \end{bmatrix}.
\]
(15)

The metrics we chose to measure the performance of the system are probability of detection and probability of
false alarm. The accurate detection of the presence or absence of PU, without any interferences, is detection and a false alarm causes a SU to miss the opportunity to use the unused channel. For local sensing the decision rule is given as

\[
z_i < \gamma_i \quad H_0
\]

\[
z_i > \gamma_i \quad H_1
\]

where, \( \gamma_i \) is the threshold for decision for SU \( i \).

Based on the above threshold the probability of false alarm according to [4] is given by

\[
P_f = P(H_1 | H_0) = Q \left( \frac{y_i - E[z_i | H_0]}{\sqrt{\text{Var}[z_i | H_0]}} \right),
\]

where \( P(H_1 | H_0) \) is the probability that \( H_1 \) is true i.e., that the PU is active while in reality \( H_0 \) is true i.e., that the PU is not active. The probability of detection can be written as

\[
P_d = P(H_1 | H_1) = Q \left( \frac{y_i - E[z_i | H_1]}{\sqrt{\text{Var}[z_i | H_1]}} \right),
\]

where \( P(H_1 | H_1) \) is the probability that \( H_1 \) is true, i.e., the PUs are active and actually \( H_1 \) is true, i.e., the PU is active. By some changes in equation (17) we can write as

\[
y_1 = \sqrt{\text{Var}[z_i | H_0]} Q^{-1}(P_f) + E[z_i | H_0].
\]

(19)

Substituting (19) in (18) we can write probability of detection as

\[
P_d = Q \left( \frac{w_i^T \Sigma_{H_0} w_i Q^{-1}(P_f) - g_i^T w_i}{\sqrt{w_i^T \Sigma_{H_1} w_i}} \right),
\]

(20)

where \( g_i = (\zeta_i \sigma_i^2, R_1(1)|h_i|^2) \). Our ultimate goal is to maximize the probability of detection. From the equation (20) to maximize that, the value of \( f(w_i) \) must be minimum,

\[
\min_{w_i} f(w_i)
\]

(21)

where

\[
f(w_i) = \frac{\sqrt{w_i^T \Sigma_{H_0} w_i Q^{-1}(P_f) - g_i^T w_i}}{\sqrt{w_i^T \Sigma_{H_1} w_i}},
\]

(22)

It is to be noted that the threshold for decision directly relates to the weighted vector \( w_i \) for a given probability of false alarm (P\( f \)). For the decision threshold to be optimum, equation (21) is to be solved over weight vector \( w_i \). The optimization problem can be solved using methods in [4] and 171

\[
\Sigma_{H_0}^{-1} = \begin{bmatrix}
N \text{diag}^2(\sigma) + \text{diag}(\delta_u) + 2E \text{diag}(\sigma) \text{diag}(h) \\
2R_1(1) \text{diag}(\sigma) \text{diag}(h)
\end{bmatrix}
\]

(23)

CRs are connected to an infrastructure and, having the sensing results of all CRs, decisions are made by the centralized system for each CR. In a de-centralized system, all CRs are connected to each other and each CR has access to information about the sensing results of rest of the CRs in the network and decisions are made by individual CRs from that information.

In this type of sensing, both statistics \( u_\text{and} \ r \) are transmitted to the fusion center through a noisy control channel and a global test statistic is calculated in the fusion center for the purpose of detection. The received signal at fusion center can be written as

\[
y = \frac{y_u}{y_r} + \frac{n_u}{n_r},
\]

(23)

where \( u = (u_1, u_2, ..., u_M) \) and \( r = (r_1, r_2, ..., r_M) \). We define \( n = (n_1, n_2, n_3, ..., n_M) \), in which the variances are collected into the vector form \( \delta_u = (\delta_{u_1}, \delta_{u_2}, ..., \delta_{u_M}) \), \( n = (n_1, n_2, n_3, ..., n_M) \), in which the variances are collected into the vector form \( \delta_r = (\delta_{r_1}, \delta_{r_2}, ..., \delta_{r_M}) \).

The received signals are normal with means \( E[y_u] = E[u] \) and \( E[y_r] = E[r] \) and variances \( \text{Var}[y_u] = \text{Var}[u] + \delta_u^2 \) and \( \text{Var}[y_r] = \text{Var}[r] + \delta_r^2 \). We can write the global test statistic as a linear combination of all the received signals as

\[
y_c = w_u^T y_u + w_r^T y_r = w_f^T y,
\]

(24)

where \( w = (w_u, w_r) \) in which \( w_u = (w_{u_1}, w_{u_2}, ..., w_{u_M}) \) and \( w_r = (w_{r_1}, w_{r_2}, ..., w_{r_M}) \) are the weight vectors which represent the contribution of each received signal in the decision making. The mean of \( z_i \) is given by

\[
E[z_i] = \begin{bmatrix}
Nw_u^T \sigma^2 H_0 + Nw_r^T (\sigma^2 + E_h) + w_r^T h R_1(1, H_1)
\end{bmatrix}
\]

(25)

where \( h = (h_1, h_2, h_3, ..., h_M) \), \( \sigma^2 = \sigma_1^2, \sigma_2^2, ..., \sigma_M^2 \).

The variances under different hypotheses are given by

\[
\text{Var}[z_i | H_0] = w_f^T \Sigma_{H_0} w_f, \quad i = 0, 1.
\]

(26)

where \( \Sigma_{H_0} \) and \( \Sigma_{H_1} \) (given at the end of the page as equation 28) are given as

\[
\Sigma_{H_0} = \begin{bmatrix}
N \text{diag}^2(\sigma) + \text{diag}(\delta_u) & 0_{M \times M} \\
0_{M \times M} & (N - 1) \text{diag}^2(\sigma) + \text{diag}(\delta_r)
\end{bmatrix}
\]

(27)

In global sensing the detector can be defined as

\[
z_i < \gamma_i \quad H_0
\]

\[
z_i > \gamma_i \quad H_1
\]

Similar to the probability of detection of local sensing

\[
\begin{align}
\text{Prob. of false alarm} & = \int \int \cdots \int \int f(w_f) \text{d}w_f \\
\text{Prob. of detection} & = \int \int \cdots \int \int f(w_f) \text{d}w_f
\end{align}
\]

(29)

where \( \text{Prob. of false alarm} = (E_h, R_1(1)|h|) \). For a specified probability of false alarm, the solution for maximizing probability of detection is

\[
\min_{w_f} f(w_f)
\]

(31)
where
\[
f(w_f) = \frac{\sqrt{w_f^T \Sigma H_0 w_f} Q^{-1}(P_f) - g^T w_f}{\sqrt{w_f^T \Sigma H_1 w_f}}.
\]

This optimization problem can also be solved by the proposed methods in [4] and [7].

IV. RESULTS AND GRAPHS

In this section we simulate the scheme for different scenarios. We assumed M CR users in the network topology. For reducing the complexity we have taken the transmitted primary signal as \(s(k) = 1\). In Fig. 1, for the simulation, we considered a cognitive radio network with \(M = 6, N = 20, \sigma_i^2 = 1, \forall i = 1, \forall i\). We took the local SNRs at each CRs as \([9.3, 7.8, 9.6, 7.6, 3.5, 9.2]\) in dB. The results we achieved are based on \(10^6\) noise realizations.

For Fig. 2, the simulation parameters are \(N = 20, \delta_{\text{ui}} = \delta_{\text{ri}} = 1, \forall i\) and the graph is depicted for values of \(M = 1, 3, 6, 10\), where the corresponding values of average local SNRs are \([9.3, 6.7, 9.2, 9.0]\) in dB. For \(M = 1\), the noise level is considered as \(\sigma = 1.9\) and the local SNR is 8.3. For \(M = 3\), the noise level is \(\sigma = \{0.7, 1.0, 0.8\}\) and the local SNRs are \([10.4, 9.3, 2.6]\) dB. For \(M = 6\), the noise levels are considered as \(\sigma = \{0.9, 1.3, 1.0, 2.0, 1.8, 1.2\}\) and local SNRs are \([7.2, 5.1, 10.8, -1.2, 3.6, 9.7]\) in dB. For \(M = 10\), thenoise levels are considered as \(\sigma = \{0.9, 1.3, 1.0, 2.0, 1.8, 1.2, 1.5, 0.8, 1.1, 1.8\}\) and local SNRs are \([7.2, 5.1, 10.8, -1.2, 3.6, 9.7, 9.0, 8.4, 6.0, -0.8]\) in dB. The probabilities in Fig. 2 clearly shows that, as the number of CRs in the network increases, the probability of detection increases and the probability of false alarm decreases because of the cooperation between the CRs.

V. CONCLUSION

In this paper we have proposed a novel algorithm for cooperative spectrum sensing in cognitive radio networks to increase probability of detection and decrease probability of false alarm. We have used two methods of non-cooperative spectrum sensing, energy and first order correlation detection and, the algorithm has been formulated by the linear combination of both. The simulations show that the algorithm improves by number users in the network and is an improvement on the traditional non-cooperative methods.

REFERENCES


