

# Impact of Variable Ordering Cost and Promotional Effort Cost in Deteriorated Economic Order Quantity (EOQ) Model

Padmabati Gahan, Monalisha Pattnaik\*

Dept. of Business Administration, Sambalpur University, Jyotivihar, Burla, India

**Abstract**— The instantaneous economic order quantity (EOQ) profit optimization model for deteriorating items is introduced for analyzing the impact of variable ordering cost and promotional effort cost for leveraging profit margins in finite planning horizons. The objective of this model is to maximize the net profit so as to determine the order quantity and promotional effort factor. For any given number of replenishment cycles the existence of a unique optimal replenishment schedule are proved and further the concavity of the net profit function of the inventory system in the number of replenishments is established. The numerical analysis shows that an appropriate policy can benefit the retailer, especially for deteriorating items. Finally, sensitivity analyses with respect to the major parameters are also studied to draw managerial decisions in production systems.

**Keywords**— Variable ordering cost, Promotional effort cost, Deterioration, Profit.

## I. INTRODUCTION

Most of the literature on inventory control and production planning has dealt with the assumption that the demand for a product will continue infinitely in the future either in a deterministic or in a stochastic fashion. This assumption does not always hold true. Inventory management plays a crucial role in businesses since it can help companies reach the goal of ensuring prompt delivery, avoiding shortages, helping sales at competitive prices and so forth. The mathematical modeling of real-world inventory problems necessitates the simplification of assumptions to make the mathematics flexible. However, excessive simplification of assumptions results in mathematical models that do not represent the inventory situation to be analyzed.

Many models have been proposed to deal with a variety of inventory problems. The classical analysis of inventory control considers three costs for holding inventories. These costs are the procurement cost, carrying cost and shortage cost. The classical analysis builds a model of an inventory

system and calculates the EOQ which minimize these three costs so that their sum is satisfying minimization criterion. One of the unrealistic assumptions is that items stocked preserve their physical characteristics during their stay in inventory. Items in stock are subject to many possible risks, e.g. damage, spoilage, dryness; vaporization etc., those results decrease of usefulness of the original one and a cost is incurred to account for such risks.

The EOQ inventory control model was introduced in the earliest decades of this century and is still widely accepted by many industries today. Comprehensive reviews of inventory models can be found in Osteryoung, Mccarty and Reinhart (1986), Pattnaik (2011) and Pattnaik (2013). In previous deterministic inventory models, many are developed under the assumption that demand is either constant or stock dependent for deteriorated items. Jain and Silver (1994) developed a stochastic dynamic programming model presented for determining the optimal ordering policy for a perishable or potentially obsolete product so as to satisfy known time-varying demand over a specified planning horizon. They assumed a random lifetime perishability, where, at the end of each discrete period, the total remaining inventory either becomes worthless or remains usable for at least the next period. Mishra (2012) explored the inventory model for time dependent holding cost and deterioration with salvage value where shortages are allowed. Gupta and Gerchak (1995) examined the simultaneous selection product durability and order quantity for items that deteriorate over time. Their choice of product durability is modeled as the values of a single design parameter that effects the distribution of the time-to-onset of deterioration (TOD) and analyzed two scenarios; the first considers TOD as a constant and the store manager may choose an appropriate value, while the second assumes that TOD is a random variable. Goyal and Gunasekaran (1995) considered the effect of different marketing policies, e.g. the price per unit product and the advertisement frequency on the demand of a perishable item. Bose, Goswami and

Chaudhuri (1995) considered an economic order quantity (EOQ) inventory model for deteriorating goods developed with a linear, positive trend in demand allowing inventory shortages and backlogging. Bose, Goswami and Chaudhuri (1995) and Hariga (1996) investigated the effects of inflation and the time-value of money with the assumption of two inflation rates rather than one, i.e. the internal (company) inflation rate and the external (general economy) inflation rate. Hariga (1994) argued that the analysis of Bose, Goswami and Chaudhuri (1995) contained mathematical errors for which he proposed the correct theory for the problem supplied with numerical examples. Pattnaik (2011) explained a single item EOQ model with demand dependent unit cost and variable setup cost. Padmanabhan and Vrat (1995) presented an EOQ inventory model for perishable items with a stock dependent selling rate. They assumed that the selling rate is a function of the current inventory level and the rate of deterioration is taken to be constant. Pattnaik (2012) explained a non-linear profit-maximization entropic order quantity model for deteriorating items with stock dependent demand rate. Pattnaik (2013) introduced a fuzzy EOQ model with demand dependent unit cost and varied setup cost under limited storage capacity.

The most recent work found in the literature is that of Hariga (1995) who extended his earlier work by assuming a time-varying demand over a finite planning horizon. Pattnaik (2011) assumes instant deterioration of perishable items with constant demand where discounts are allowed. Pattnaik (2010) presented an entropic order quantity (EnOQ) model under instant deterioration for perishable items with constant demand where discounts are allowed. Salameh, Jabar and Nouhed (1999) studied an EOQ inventory model in which it assumes that the percentage of on-hand inventory wasted due to deterioration is a key feature of the inventory conditions which govern the item stocked.

Pattnaik (2011) discussed an entropic order quantity (EnOQ) model under cash discounts. Pattnaik (2012) introduced an EOQ model for perishable items with constant demand and instant deterioration. Pattnaik (2012) studied the effect of promotion in fuzzy optimal replenishment model with units lost due to deterioration. Pattnaik (2013) investigated linear programming problems in fuzzy environment with evaluating the post optimal

analyses. Pattnaik (2013) discussed wasting of percentage on-hand inventory of an instantaneous economic order quantity model due to deterioration. Raafat (1991) explained survey of literature on continuously deteriorating inventory models. Roy and Maiti (1997) presented fuzzy EOQ model with demand dependent unit cost under limited storage capacity. Tripathy, Pattnaik and Tripathy (2012) introduced optimal EOQ model for Deteriorating Items with Promotional Effort Cost. Tripathy, Pattnaik and Tripathy (2013) presented a decision-making framework for a single item EOQ model with two constraints. Tsao and Sheen (2008) explored dynamic pricing, promotion and replenishment policies for a deteriorating item under permissible delay in payment. Waters (1994) and Pattnaik (2012) defined various inventory models with managerial decisions. Wee (1993) explained an economic production lot size model for deteriorating items with partial back-ordering. In this model, replenishment decision under none wasting the percentage of on-hand inventory due to deterioration are adjusted arbitrarily upward or downward for profit maximization model in response to the change in market demand within the finite planning horizon with dynamic setup cost with promotional effort cost. The objective of this model is to determine optimal replenishment quantities and optimal promotional effort factor in an instantaneous replenishment profit maximization model.

All mentioned above inventory literatures with deterioration has the basic assumption that the retailer owns a storage room with optimal order quantity. In recent years, companies have started to recognize that a tradeoff exists between product varieties in terms of quality of the product for running in the market smoothly. In the absence of a proper quantitative model to measure the effect of product quality of the product, these companies have mainly relied on qualitative judgment. This model postulates that measuring the behavior of production systems may be achievable by incorporating the idea of retailer in making optimum decision on replenishment with the percentage of on-hand inventory due to deterioration is not lost with dynamic ordering cost and then compares the optimal results with fixed ordering cost in traditional model. The major assumptions used in the above research articles are summarized in Table1.

Table.1: Summary of the Related Researches

Author(s) and published Year	Structure of the model	Demand	Demand patterns	Deterioration	Setup Cost	Units lost	Planning	Model
Hariga (1994)	Crisp (EOQ)	Time	Non-stationary	Yes	Constant	No	Finite	Cost
Tsao et al. (2008)	Crisp (EOQ)	Time and Price	Linear and Decreasing	Yes	Constant	No	Finite	Profit
Pattnaik (2009)	Crisp (EnOQ)	Constant (Deterministic)	Constant	Yes (Instant)	Constant	No	Finite	Profit
Pattnaik (2011)	Crisp (EOQ)	Constant (Deterministic)	Constant	Yes (Instant)	Constant	No	Finite	Profit
Salameh et al. (1993)	Crisp (EOQ)	Constant (Deterministic)	Constant	Yes	Constant	Yes	Finite	Profit
Present Paper (2016)	Crisp (EOQ)	Constant (Deterministic)	Constant	Yes	Variable	No	Finite	Profit

The remainder of the model is organized as follows. In Section 2 assumptions and notations are provided for the development of the model. The mathematical formulation is developed in Section 3. The solution procedure is given in Section 4. In Section 5, numerical example is presented to illustrate the development of the model. The sensitivity analysis is carried out in Section 6 to observe the changes in the optimal solution. Finally Section 7 deals with the summary and the concluding remark.

## II. ASSUMPTIONS AND NOTATIONS

- r Consumption rate,
- $t_c$  Cycle length,
- h Holding cost of one unit for one unit of time,
- HC (q,ρ) Holding cost per cycle,
- c Purchasing cost per unit,
- $P_s$  Selling Price per unit,
- $\alpha$  Percentage of on-hand inventory that is lost due to deterioration,
- q Order quantity,
- $K \times (q^{\gamma-1})$  Ordering cost per cycle where,  $0 < \gamma < 1$ ,
- $q^*$  Traditional economic ordering quantity (EOQ),
- $\varphi(t)$  On-hand inventory level at time t,
- $\rho$  The promotional effort factor per cycle,
- PE(ρ) The promotional effort cost,  $PE(\rho) = K_1(\rho - 1)^2 \gamma^{\alpha_1}$  where,  $K_1 > 0$  and  $\alpha_1$  is a constant,
- $\pi_1(q, \rho)$  Net profit per unit of producing q units per cycle in crisp strategy,

$\pi(q, \rho)$  Average profit per unit of producing q units per cycle in crisp strategy,

## III. MATHEMATICAL MODEL

Denote  $\varphi(t)$  as the on-hand inventory level at time t. During a change in time from point t to t+dt, where  $t + dt > t$ , the on-hand inventory drops from  $\varphi(t)$  to  $\varphi(t+dt)$ . Then  $\varphi(t+dt)$  is given as:

$$\varphi(t+dt) = \varphi(t) - r \rho dt - \alpha \varphi(t) dt$$

$$\varphi(t+dt) \text{ can be re-written as: } \frac{\varphi(t+dt) - \varphi(t)}{dt} = -r\rho - \alpha\varphi(t)$$

and  $dt \rightarrow 0$ , the above equation reduces to:  $\frac{d\varphi(t)}{dt} + \alpha\varphi(t) + r\rho = 0$

It is a differential equation, solution is

$$\varphi(t) = \frac{-r\rho}{\alpha} + \left(q + \frac{r\rho}{\alpha}\right) \times e^{-\alpha t}$$

Where q is the order quantity which is instantaneously replenished at the beginning of each cycle of length  $t_c$  units of time. The stock is replenished by q units each time these units are totally depleted as a result of outside demand and deterioration. Behavior of the inventory level for the above model is illustrated in Fig. 1. The cycle length,  $t_c$ , is determined by first substituting  $t_c$  into equation  $\varphi(t)$  and then setting it equal to zero to get:  $t_c = \frac{1}{\alpha} \ln \left( \frac{\alpha q + r\rho}{r\rho} \right)$

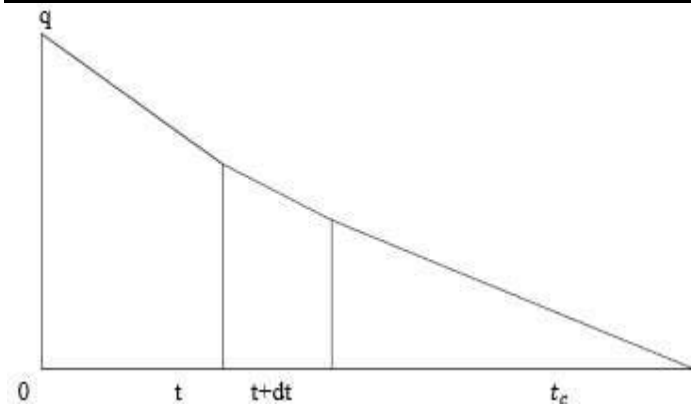


Fig. 1: Behavior of the Inventory over a Cycle for a Deteriorating Item

Equation  $\phi(t)$  and  $t_c$  are used to develop the mathematical model. It is worthy to mention that as  $\alpha$  approaches to zero,  $t_c$  approaches to  $\frac{q}{r\rho}$ . The total cost per cycle,  $TC(q, \rho)$ , is the sum of the variable ordering cost and purchasing cost per cycle,  $Kq^{(\gamma-1)} + cq$ , the holding cost per cycle,  $HC(q, \rho)$ , and the promotional effort cost per cycle,  $PE(\rho)$ .  $HC(q, \rho)$  is obtained from equation  $\phi(t)$  as:

$$HC(q, \rho) = \int_0^{t_c} h\phi(t)dt = h \int_0^{\frac{1}{\alpha} \ln\left(\frac{\alpha q + r\rho}{r\rho}\right)} \left[-\frac{r\rho}{\alpha} + \left(q + \frac{r\rho}{\alpha}\right) \times e^{-\alpha t}\right] dt$$

$$= h \times \left[\frac{q}{\alpha} - \frac{r\rho}{\alpha^2} \ln\left(\frac{\alpha q + r\rho}{r\rho}\right)\right]$$

$$PE(\rho) = K_1(\rho - 1)^2 r^{\alpha_1}$$

$$TC = OC + PC + HC + PE$$

$$TC(q, \rho) = Kq^{(\gamma-1)} + cq + h \times \left[\frac{q}{\alpha} - \frac{r\rho}{\alpha^2} \ln\left(\frac{\alpha q + r\rho}{r\rho}\right)\right] + K_1(\rho - 1)^2 r^{\alpha_1}$$

The total cost per unit of time,  $TCU(q, \rho)$ , is given by dividing equation  $TC(q, \rho)$  by equation  $t_c$  to give:

$$TCU(q, \rho) = \left[ Kq^{(\gamma-1)} + cq + h \times \left[\frac{q}{\alpha} - \frac{r\rho}{\alpha^2} \ln\left(\frac{\alpha q + r\rho}{r\rho}\right)\right] + K_1(\rho - 1)^2 r^{\alpha_1} \right] \times \left[\frac{1}{\alpha} \ln\left(\frac{\alpha q + r\rho}{r\rho}\right)\right]^{-1}$$

$$= \frac{Kq^{(\gamma-1)\alpha + (c\alpha + h)q}}{\ln\left(1 + \frac{\alpha q}{r\rho}\right)} - \frac{hr\rho}{\alpha} + \frac{K_1\alpha(\rho-1)^2 r^{\alpha_1}}{\ln\left(1 + \frac{\alpha q}{r\rho}\right)}$$

As  $\alpha$  approaches zero and  $\rho = 1$  equation  $TCU(q, \rho)$  reduces to  $TCU(q) = \frac{Kq^{(\gamma-1)r}}{q} + cr + \frac{hq}{2}$ . Whose solution is given by

$$\text{the traditional EOQ formula, } q^* = \left[\frac{h}{2Kr(2-\gamma)}\right]^{1/\gamma-3}$$

The total profit per cycle with  $\alpha$  approaching to zero only is  $\pi_1(q, \rho)$ .  $\pi_1(q, \rho) = q \times P_s - TC(q, \rho) = qP_s - Kq^{(\gamma-1)} - cq - \frac{hq^2}{2r\rho} - K_1(\rho - 1)^2 r^{\alpha_1}$

$TC(q, \rho)$  the total cost per cycle, are calculated from equation  $TC(q, \rho)$ . Whose solution is given by the traditional EOQ formula,  $q^* = \left[\frac{h}{2Kr\rho(2-\gamma)}\right]^{1/\gamma-3}$ . The average profit  $\pi(q, \rho)$  per unit time is obtained by dividing  $t_c$  in  $\pi_1(q, \rho)$ . Hence the profit maximization problem is  
 Maximize  $\pi_1(q, \rho)$   
 $\forall q \geq 0, \rho \geq 0$

#### IV. OPTIMIZATION

The optimal ordering quantity  $q$  and promotional effort  $\rho$  per cycle can be determined by differentiating equation  $\pi_1(q, \rho)$  with respect to  $q$  and  $\rho$  separately, setting these to zero.

In order to show the uniqueness of the solution in, it is sufficient to show that the net profit function throughout the cycle is jointly concave in terms of ordering quantity  $q$  and promotional effort factor  $\rho$ . The second partial derivatives of equation  $\pi_1(q, \rho)$  with respect to  $q$  and  $\rho$  are strictly negative and the determinant of Hessian matrix is positive. Considering the following propositions:

**Proposition 1** The net profit  $\pi_1(q, \rho)$  per cycle is concave in  $q$ .

Conditions for optimal  $q$

$$\frac{\partial \pi_1(q, \rho)}{\partial q} = P_s - \left(K(\gamma - 1)q^{\gamma-2} + c + \frac{hq}{r\rho}\right) = 0$$

The second order partial derivative of the net profit per cycle with respect to  $q$  can be expressed as:

$$\frac{\partial^2 \pi_1(q, \rho)}{\partial q^2} = -\frac{h}{r\rho} - (K(\gamma - 1)(\gamma - 2)q^{\gamma-3}),$$

Since  $r\rho > 0$ ,  $(\gamma - 1)(\gamma - 2) > 0$  and  $h > 0$  Equation  $\frac{\partial^2 \pi_1(q, \rho)}{\partial q^2}$  is negative.

**Proposition 2** The net profit  $\pi_1(q, \rho)$  per cycle is concave in  $\rho$ .

Conditions for optimal  $\rho$

$$\frac{\partial \pi_1(q, \rho)}{\partial \rho} = \left(\frac{hq^2}{2r\rho^2}\right) - 2K_1(\rho - 1)r^{\alpha_1} = 0$$

The second order partial derivative of the net profit per cycle with respect to  $\rho$  is

$$\frac{\partial^2 \pi_1(q, \rho)}{\partial \rho^2} = -\frac{hq^2}{r\rho^3} - 2K_1r^{\alpha_1}$$

Since  $\left(\frac{hq^2}{r\rho^3}\right) > 0$ ,  $K_1 > 0$ ,  $r > 0$ , it is found that  $\frac{\partial^2 \pi_1(q, \rho)}{\partial \rho^2}$  is negative.

Propositions 1 and 2 show that the second partial derivatives of equation  $\pi_1(q, \rho)$  with respect to  $q$  and  $\rho$

separately are strictly negative. The next step is to check that the determinant of the Hessian matrix is positive, i.e.

$$\frac{\partial^2 \pi_1(q, \rho)}{\partial q^2} \times \frac{\partial^2 \pi_1(q, \rho)}{\partial \rho^2} - \left( \frac{\partial^2 \pi_1(q, \rho)}{\partial q \partial \rho} \right)^2 > 0 = \frac{2h}{r\rho} K_1 r^{\alpha_1} + K(\gamma - 1)(\gamma - 2)q^{\gamma-3} \frac{hq^2}{r\rho^3} + 2K_1 r^{\alpha_1} K(\gamma - 1)(\gamma - 2)q^{\gamma-3} > 0,$$

Since

$$\left( \frac{\partial^2 \pi_1(q, \rho)}{\partial q^2} \right), \left( \frac{\partial^2 \pi_1(q, \rho)}{\partial \rho^2} \right) \text{ shown in } \frac{\partial \pi_1(q, \rho)}{\partial q} \text{ and } \frac{\partial \pi_1(q, \rho)}{\partial \rho} \text{ and } \frac{\partial^2 \pi_1(q, \rho)}{\partial q \partial \rho} = \frac{\partial^2 \pi_1(q, \rho)}{\partial \rho \partial q} = \frac{hq}{r\rho^2}$$

The objective is to determine the optimal values of q and ρ to maximize the net profit function. It is very difficult to derive the optimal values of q and ρ, hence unit profit function. There are several methods to cope with constraints optimization problem numerically. But here LINGO 13.0 software is used to derive the optimal values of the decision variables.

**V. NUMERICAL EXAMPLE**

Consider an inventory situation where K is Rs. 200 per order, h is Rs. 5 per unit per unit of time, r is 1200 units per unit of time, c is Rs. 100 per unit, the selling price per unit P<sub>s</sub> is Rs. 125, γ is 0.5 and α is 0%, K<sub>1</sub> = 2.0 and α<sub>1</sub> = 1.0. The optimal solution that maximizes equation π<sub>1</sub>(q, ρ) and q\*\* and ρ\* are determined by using LINGO 13.0 version software and the results are tabulated in Table 2. In the present model the net profit, units lost due to deterioration, the cycle length and order quantity are comparatively more than that of the comparative models, it indicates the present model incorporated with promotional effort cost and variable ordering cost may draw the better decisions in managerial uncertain space. Fig. 2 represents the relationship between the order quantity q and dynamic setup cost OC. Fig. 3 represents the three dimensional mesh plot order quantity q, promotional effort factor ρ and net profit per cycle π<sub>1</sub>. Fig. 4 is the sensitivity plotting of order quantity q, promotional effort factor ρ and net profit per cycle π<sub>1</sub>(q, ρ).

Table.2: Optimal Values of the Proposed Model

Model	Iteration	t*	L*	q*	ρ*	OC	PE	π <sub>1</sub> (q)	π(q)
Crisp	92	5.000001	-	99750.04	16.625	0.6332475	585937.8	660936.9	132187.4
Crisp	417	5.000043	-	6000.052	-	2.59	-	74997.42	14999.35501
Crisp	41	0.258	-	309.839	-	-	-	7345.9678	28450.81

**VI. SENSITIVITY ANALYSIS**

It is interesting to investigate the influence of the major parameters K, h, r, c, P<sub>s</sub>, γ, K<sub>1</sub> and α<sub>1</sub> on retailer's behaviour. The computational results shown in Table 11.5.4 indicate the following managerial phenomena:

- t<sub>c</sub> the replenishment cycle length, q the optimal replenishment quantity, ρ the optimal promotional effort factor, PE promotional effort cost, π<sub>1</sub> the optimal net profit per unit per cycle and π the optimal average profit per unit per cycle are insensitive to the parameter K but OC variable setup cost is sensitive to the parameter K.
- t<sub>c</sub> the replenishment cycle length, q the optimal replenishment quantity, ρ the optimal promotional effort factor, PE promotional effort cost, OC variable setup cost, π<sub>1</sub> the optimal net profit per unit per cycle and π the optimal average profit per unit per cycle are sensitive to the parameter h.
- t<sub>c</sub> the replenishment cycle length and ρ the optimal promotional effort factor and OC variable setup cost is moderately insensitive to the parameter r but q the optimal replenishment quantity, PE promotional effort cost, π<sub>1</sub> the optimal net profit per unit per cycle and π the

optimal average profit per unit per cycle are sensitive to the parameter r

- t<sub>c</sub> the replenishment cycle length, q the optimal replenishment quantity, ρ the optimal promotional effort factor, PE promotional effort cost, OC variable setup cost, π<sub>1</sub> the optimal net profit per unit per cycle and π the optimal average profit per unit per cycle are sensitive to the parameter c.
- t<sub>c</sub> the replenishment cycle length, q the optimal replenishment quantity, ρ the optimal promotional effort factor, PE promotional effort cost, OC variable setup cost, π<sub>1</sub> the optimal net profit per unit per cycle and π the optimal average profit per unit per cycle are sensitive to the parameter P<sub>s</sub>.
- t<sub>c</sub> the replenishment cycle length and ρ the optimal promotional effort factor, q the optimal replenishment quantity, PE promotional effort cost, π<sub>1</sub> the optimal net profit per unit per cycle and π the optimal average profit per unit per cycle are insensitive to the parameter γ and OC variable setup cost is sensitive to the parameter γ.
- t<sub>c</sub> the replenishment cycle length is insensitive to the parameter K<sub>1</sub> but ρ the optimal promotional effort factor,

q the optimal replenishment quantity, OC variable setup cost, PE promotional effort cost,  $\pi_1$  the optimal net profit per unit per cycle and  $\pi$  the optimal average profit per unit per cycle are sensitive to the parameter  $K_1$ .

- $t_c$  the replenishment cycle length is insensitive to the parameter  $\alpha_1$  but  $\rho$  the optimal promotional effort factor,

q the optimal replenishment quantity, OC variable setup cost, PE promotional effort cost,  $\pi_1$  the optimal net profit per unit per cycle and  $\pi$  the optimal average profit per unit per cycle are sensitive with static to the parameter  $\alpha_1$ .

Table.3: Sensitivity Analyses of the parameters K, h, r, c,  $P_s$ ,  $\gamma$ ,  $K_1$  and  $\alpha_1$

Parameter	Value	Iteration	$t^*$	$q^*$	$\rho^*$	OC	PE	$\pi_1(q, \rho)$	$\pi(q, \rho)$
K	150	83	5	99750.03	16.625	0.474934	585937.7	660937	132187.4
	250	90	5.000001	99750.05	16.625	0.79155	585937.9	660936.7	132187.3
	500	85	5.000001	99750.05	16.625	0.94987	585937.9	660936.6	132187.5
h	3	59	8.333334	270416.7	27.04167	0.3846	1627604	1752604	210312.4
	8	120	3.125002	40371.15	10.76563	0.99539	228882.3	275755.8	88241.83
	10	100	2.500002	26437.57	8.812515	1.23004	146484.9	183983.1	73593.19
r	1250	88	5.000001	103906.3	16.625	0.62045	610351.9	688475.9	137695.2
	1300	80	5.000001	108062.5	16.625	0.60840	634765.9	716015.0	143203
	1400	110	5.000001	116375	16.625	0.58627	683594	771093.2	154218.6
c	105	79	4.400001	69168.05	13.10001	0.76046	351384.4	409463.2	93059.8
	108	81	4.000002	52800.06	11.00001	0.87039	240000.4	287999.1	71999.75
	103	72	3.400003	33558.09	8.225014	1.09177	125282	159960.4	47047.13
$P_s$	120	76	4.000002	52800.06	11.00001	0.87039	240000.4	287999.1	71999.5
	130	82	6	169200	23.5	0.48622	1215000	1323000	220499.9
	132	86	6.4	204288	26.6	0.44250	1572864	1695744	264959.9
$\gamma$	0.1	89	5	99750	16.625	0.00633	585937.5	660937.5	132187.5
	0.3	72	5	99750.01	16.625	0.06336	585937.5	660937.4	132187.5
	0.6	96	5.000002	99750.09	16.62501	2.00200	585938.3	660935.5	132187.1
$K_1$	3	76	5.000001	68500.04	11.41667	0.76416	890625.3	465624.2	93124.83
	5	95	5.000002	43500.05	7.250006	0.95893	234375.4	309374	61874.78
	10	103	5.000005	14750.06	4.125006	1.27128	117188	192186.2	38437.21
$\alpha_1$	2	106	5.000042	6078.178	1.013021	2.56533	488.2977	75485.72	15097.02
	3	92	5.000043	6000.117	1.000011	2.58196	0.4069150	74997.82	14999.44
	4	85	5.000043	6000.052	1	2.58198	0.00339095	74997.42	14999.35

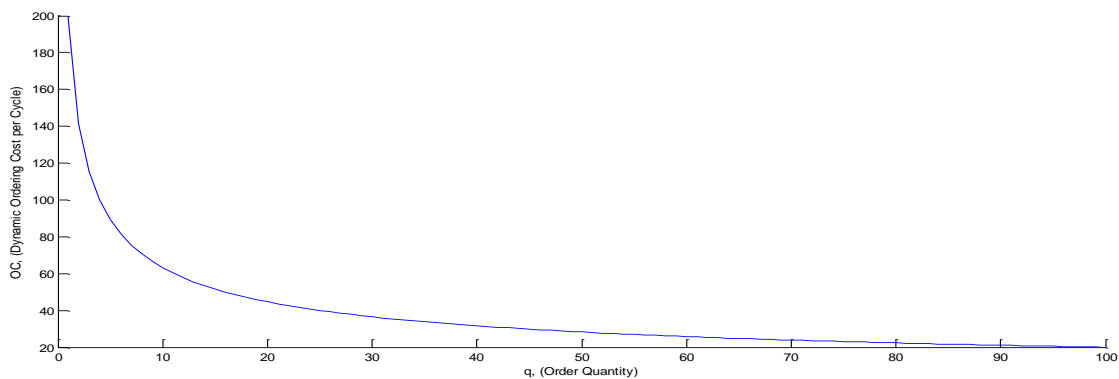


Fig.2: Two dimensional plot of Order Quantity, q and Dynamic Ordering Cost, OC

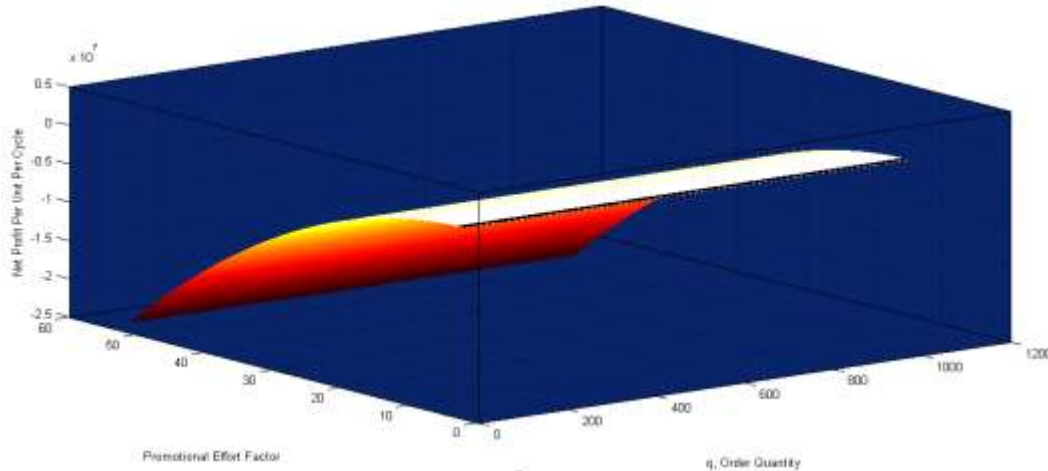


Fig.3: Three Dimensional Mesh Plot of Order Quantity  $q$ , Promotional Effort Factor  $\rho$  and Net Profit per Cycle  $\pi_1(q, \rho)$

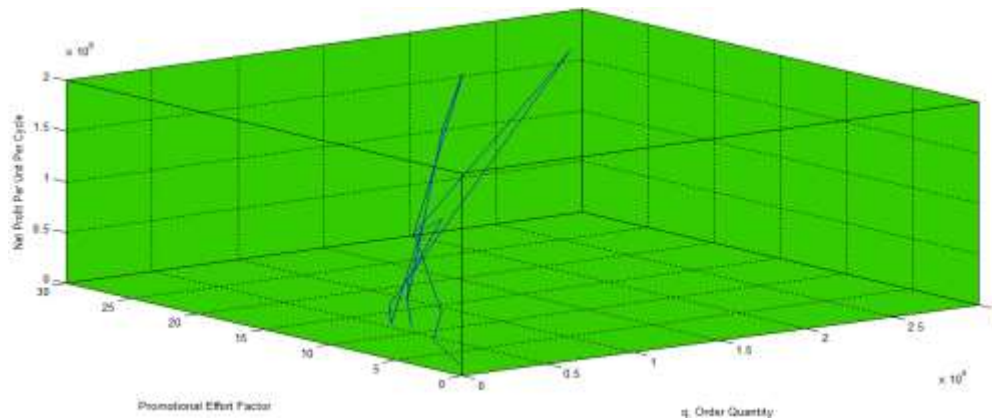


Fig. 4: Sensitivity Plotting of Order Quantity  $q$ , Promotional Effort Factor  $\rho$  and Net Profit per Cycle  $\pi_1(q, \rho)$

## VII. CONCLUSION

In this model, it investigates the optimal order quantity which assumes that a percentage of the on-hand inventory is not wasted due to deterioration for variable setup cost characteristic features and the inventory conditions govern the item stocked. This model provides a useful property for finding the optimal profit and ordering quantity for deteriorated items. A new mathematical model with dynamic setup cost is developed and compared to the traditional EOQ model numerically. The economic order quantity,  $q^*$  and the net profit for the modified model, were found to be more than that of the traditional,  $q$ , i.e.  $q^* > q$  and the net profit respectively. The modified average profit per unit per cycle is more than that of the traditional average profit per unit per cycle. Hence the utilization of variable setup cost makes the scope of the application broader. Further, a numerical example is presented to illustrate the theoretical results, and some observations are obtained from sensitivity analyses with respect to the major parameters.

The model in this study is a general framework that considers variable setup cost without wasting the percentage of on-hand inventory due to deterioration simultaneously.

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