Multi-Index Bi-Criterion Transportation Problem: A Fuzzy Approach

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Abstract—*This paper represents a non linear bi-criterion generalized multi-index transportation problem (BGMTP) is considered. The generalized transportation problem (GTP) arises in many real-life applications. It has the form of a classical transportation problem, with the additional assumption that the quantities of goods change during the transportation process. Here the fuzzy constraints are used in the demand and in the budget. An efficient new solution procedure is developed keeping the budget as the first priority. All efficient time-cost trade-off pairs are obtained. D_1-distance is calculated to each trade-off pair from the ideal solution. Finally optimum solution is reached by using D_1-distance.*

Keywords— *Time-cost trade-off pair, D_1-distance, ideal solution, membership function, priority.*

I. INTRODUCTION

The cost minimizing classical multi-index transportation problems play important rule in practical problems. The cost minimizing classical multi-index transportation problems have been studied by several authors [14, 15, 16, 17] etc. Some times there may exist emergency situation eg police services, time services, hospital management etc. where time of transportation is of greater importance than cost of transportation. In this situation, it is to be noted that the cost as well as time play prominent roles to obtain the best decision. Here the two aspects (ie cost and time) are conflicting in nature. In general one can not simultaneously minimize both of them. Bi-criterion transportation problem have been studied by several authors [3, 4, 8, 17, 11] etc.

There are many business problems, industrial problems, machine assignment problems, routing problems, etc. that have the characteristic in common with generalized transportation problem that have been studied by several authors [1, 2, 4, 5, 9, 10, 14] etc.

In real world situation, most of the intimations are imprecise in nature involving vagueness or to say fuzziness. Precise mathematical model are not enough to tackle all practical problems. Fuzzy set theory was developed for solving the imprecise problems in the field of artificial intelligence. To tackle this situation fuzzy set theory are used. In this field area pioneer work came from Bellman and Zadeh [6]. Fuzzy transportation problem have been studied by several authors [12, 18, 19, 20, 21, 23, 24] etc.

The importance of fuzzy generalized multi-index transportation problem is increasing in a great deal but the method for finding time-cost trade-off pair in a bi-criterion fuzzy generalized multi-index transportation problem has been paid less attention. In this paper, we have developed a new algorithm to find time-cost trade-off pair of bi-criterion fuzzy generalized multi-index transportation problem. Thereafter an optimum time-cost trade-off pair has been obtained.

Problem Formulation:

Let there be m-origins, n-destinations and q-products in a bi-criterion generalized multi-index fuzzy transportation problem.

Let,

x_{ijk} = the amount of the k-th type of product transported from the i-th origin to the j-th destination,

t_{ijk} = the time of transporting the k-th type of product from the i-th origin to the j-th destination which is independent of amount of commodity transported so long as $x_{ijk} > 0$,

r_{ijk} = the cost involved in transporting per unit of the k-th type of product from the i-th origin to the j-th destination,

a_i = number of units available at origin i,

b_j = number of units required at the destination j,

c_k = requirements of the number of units of the k-th type of product and

d_{ijk}^1 , d_{ijk}^2 = positive constants rather than unity, due to

generalized multi-index transportation problem (GMTP).

Then the cost minimizing fuzzy GMTP can be formulated as follows:

$$P_1 : Find \ x_{ijk} \ (\geq 0) \ ; \ (1 \leq i \leq m, \ 1 \leq j \leq n, \ 1 \leq k \leq q)$$

subject to the constraints,

$$\sum_{j=1}^{n} \sum_{k=1}^{q} d_{ijk}^1 x_{ijk} \leq a_i \ ; \quad 1 \leq i \leq m,$$

$$b_j^* \lessapprox \sum_{i=1}^{m} \sum_{k=1}^{q} x_{ijk} \lessapprox b_j \ ; \quad 1 \leq j \leq n$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} d_{ijk}^2 x_{ijk} \leq c_k \ ; \quad 1 \leq k \leq q.$$

$$and \quad \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{q} r_{ijk} x_{ijk} \leq Z_1$$

$$(1)$$

Some times there may arise emergency situation, eg, hospital managements, fire services, police services etc., where the time of transportation is of greater importance than that of cost. Then time minimizing transportation problem arises. The time minimizing transportation problem can be written as:

$$P^1 : Min \ T = \max_{\substack{1 \leq i \leq m \\ 1 \leq j \leq n \\ 1 \leq k \leq q}} \left\{ t_{ijk} : x_{ijk} > 0 \right\}$$

Subject to the constraints (1).
Combining the problem P_1 and P^1, the fuzzy BGMTP appears as:

$$P : Find \ \max_{\substack{1 \leq i \leq m \\ 1 \leq j \leq n \\ 1 \leq k \leq q}} \{ t_{ijk} : x_{ijk} > 0 \ and \ x_{ijk} \ satisfies constraints (1) \}$$

subject to the constraints (1).

Difference between Classical Multi-index Transportation Problem (MTP) and Generalized Multi-index Transportation Problem (GMTP):

There are several important differences between classical MTP and GMTP which are given below:

(i) The rank of the co-efficient matrix $[x_{ijk}]_{m \times n \times q}$ is in general $m + n + q$ rather than $m + n + q - 2$, ie, all the constraints are in general independent.

(ii) In GMTP the value of x_{ijk} may not be integer, though it is integer in classical MTP.

(iii) The activity vector in GMTP is

$$P_{ijk} = d_{ijk}^1 e_i + e_{m+j} + d_{ijk}^2 e_{m+n+k}$$

Whereas in classical MTP it is

$$P_{ijk} = e_i + e_{m+j} + e_{m+n+k}.$$

(iv) In GMTP it need not be true that cells corresponding to a basic solution form a tree. Or in other words vectors in the loop are linearly independent. But in classical MTP vectors in the loop are linearly dependent.

The problem consists of two parts,

P_1 : the problem of solving the fuzzy GMTP

P^1 : the problem of minimizing the time.

To solve the problem P, the following technique is used.

The triangular membership function for the fuzzy demand constraints are

$$\mu D_j(x) = \begin{cases} \dfrac{\sum_{i=1}^{m} \sum_{k=1}^{q} x_{ijk} - (B_j^* - \Delta b_j)}{\Delta b_j} & ; if \ B_j^* - \Delta b_j \leq \sum_{i=1}^{m} \sum_{k=1}^{q} x_{ijk} < B_j^* ; \\\\ \dfrac{(B_j^* + \Delta b_j) - \sum_{i=1}^{m} \sum_{k=1}^{q} x_{ijk}}{\Delta b_j} & ; if \ B_j^* < \sum_{i-1}^{m} \sum_{k=1}^{q} x_{ijk} \leq B_j^* + \Delta b_j ; \\\\ 0 & ; if \ \sum_{i=1}^{m} \sum_{k=1}^{q} x_{ijk} > B_j^* + \Delta b_j \ and \\\\ & \quad \sum_{i=1}^{m} \sum_{k=1}^{q} x_{ijk} < B_j^* - \Delta b_j \end{cases} \quad (2)$$

where b_j^* and b_j are the lower tolerance limit and upper tolerance limit respectively of the demand j ($1 \leq j \leq n$) and

$$\Delta b_j = \frac{b_j - b_j^*}{2}, \quad B_j^* = \frac{b_j + b_j^*}{2}$$

The linear membership function of the fuzzy budget goal can be written as:

$$\mu R(x) = \begin{cases} 1 & ; \quad \text{if } \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{q} r_{ijk} x_{ijk} \leq Z_1, \\[2em] \dfrac{Z^* - \sum_{i=1}^{m}\sum_{j}^{n}\sum_{k=1} r_{ijk} x_{ijk}}{\Delta Z} & ; \quad \text{if } Z_1 < \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{q} r_{ijk} x_{ijk} < Z^*, \\[2em] 0 & ; \quad \text{if } \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{q} r_{ijk} x_{ijk} \geq Z^* \end{cases} \qquad (3)$$

Where Z^* is the upper tolerance limit of the budget goal and $\Delta Z = Z^* - Z_1$.

II. SOLUTION PROCEDURE

The fuzzy programming model of problem P_1 is equivalent to the following linear programming problem as:

Max λ

subject to the constraints

$$\left. \begin{array}{ll} \sum_{j=1}^{n}\sum_{k=1}^{q} d_{ijk}^1 x_{ijk} \leq a_i & ; \quad 1 \leq i \leq m, \\[1.5em] \sum_{i=1}^{m}\sum_{j=1}^{n} d_{ijk}^2 x_{ijk} \leq c_k & ; \quad 1 \leq k \leq q, \\[1.5em] \lambda \leq \mu D_j(x) & ; \quad 1 \leq j \leq n, \\[0.5em] \lambda \leq \mu R(x), \\[0.5em] \lambda = \min[\, 1 \leq j \leq n, \, \mu D_j(x), \, \mu R(x)] \\[0.5em] \text{and } \lambda \in [0,1] \end{array} \right\} \qquad (4)$$

After solving the problem the optimum solution X_1^* and the corresponding optimum cost Z_1^* at the first iteration are obtained. Next the problem P^1 is solved for minimizing the time.

Let $T_1^* = Min\left[\underset{\substack{1 \leq i \leq m \\ 1 \leq j \leq n \\ 1 \leq k \leq q}}{Max}\left\{ t_{ijk} : x_{ijk} > 0, x_{ijk} \in X_1^* \right\} \right]$

subject to the constraints (1)

So, for the first iteration the time-cost trade-off pair is (Z_1^*, T_1^*). Using re-optimizing technique and replacing Z_r by Z_{r+1}, $(1 \leq r \leq h-1)$ where $Z_r < Z_{r+1} < Z^*$; $\forall$ r and $1 \leq r \leq h - 1$. All efficient time-cost trade-off pairs are obtained as:

$(Z_1^*, T_1^*), (Z_2^*, T_2^*), \ldots\ldots\ldots\ldots, (Z_r^*, T_r^*), \ldots\ldots\ldots, (Z_h^*, T_h^*)$
 (say)

where $Z_1^* < Z_2^* < \ldots\ldots < Z_r^* < \ldots\ldots < Z_h^*$

and $\quad T_1^* > T_2^* > \ldots\ldots > T_r^* > \ldots\ldots > T_h^*$

The pair (Z_1^*, T_h^*) is termed as the ideal solution.

Let,

$$\left. \begin{array}{l} d_r = Z_r^* - Z_1^* \\[0.8em] \mathbf{d_{h+r}} = T_r^* - T_h^* \end{array} \right\}$$
$1 \leq r \leq h$ (5)

So,

$$(D_1)_{\text{opt}} = \underset{1\le r\le h}{\text{Min}}\,(D_1)_r$$

$$= \underset{1\le r\le h}{Min}(d_r + d_{h+r})$$

$$= d_s + d_{h+s} \quad \text{(say)}$$

$$= (D_1)_s$$

Since equal priority to cost as well as time is given, so (Z_s^*, T_s^*) attains the optimum trade-off pair.

The Algorithm:

Step - 1: Set b = 1, where b is the number of iteration.

Step - 2: Solve problem P_1. Let Z_1^* be the optimum total cost corresponding to the optimum solution X_1^*.

Step - 3: Find T_1^*

where,

$$T_1^* = \underset{\substack{1\le i\le m \\ 1\le j\le n \\ 1\le k\le q}}{Max}\{\, t_{ijk} : x_{ijk} > 0 \text{ according to } X_1^* \,\}$$

Then (Z_1^*, T_1^*) is called time-cost trade-off pair at the first iteration.

Step - 4: Define

$$r_{ijk}^{b+1} = \begin{cases} M & \text{if} & t_{ijk} \ge T_b^* \\ r_{ijk} & \text{if} & t_{ijk} < T_b^* \end{cases}$$

and $Z_{b+1} < \sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n}\sum\limits_{k=1}^{q} r_{ijk}^{b+1} x_{ijk} < Z^*$

Where $Z_{b+1} > Z_b$

Where M is a sufficiently large positive number. Let P_{b+1} be the fuzzy GMTP with the cost values r_{ijk}^{b+1}, Z_{b+1} is the aspiration level of cost and other constraints are same as in (1).

Step - 5: Find optimum solution of the problem P_{b+1}. Let Z_{b+1}^* be the total cost of problem P_{b+1}.

Step - 6: If $Z_{b+1}^* \ge M$, the algorithm terminates and go to step 8 if b + 1 > 2 otherwise go to step 10.

Otherwise in (b+1)th iteration the time-

cost trade-off pair is (Z_{b+1}^*, T_{b+1}^*).

Obviously $Z_{b+1}^* > Z_b^*$ and $T_{b+1}^* < T_b^*$.

Step - 7: Set b = b + 1 and go to step 4.

Step - 8: Let after the h-th step the algorithm terminates, ie, $Z_{h+1}^* \ge M$, then the complete set of time-cost trade-off pairs,

$$(Z_1^*, T_1^*), (Z_2^*, T_2^*), \ldots\ldots\ldots\ldots, (Z_r^*, T_r^*), \ldots\ldots\ldots\ldots, (Z_h^*, T_h^*)$$

is identified.

Among the trade-off pairs (Z_1^*, T_h^*) is recognized as the ideal solution.

Step - 9: Find $(D_1)_{opt} = \underset{1\le r\le h}{Min}(D_1)_r$

$$= \underset{1\le r\le h}{Min}(d_r + d_{h+r})$$

$$= d_s + d_{h+s} \quad \text{(say)}$$

Then (Z_s^*, T_s^*) offers the best compromise solution.

Step - 10: If $Z_2 \ge M$, ie, if h = 1, then Z_1^* is the absolute minimum cost and T_1^* is the absolute minimum time for the optimum transportation plan.

Numerical Examples:

A manufacturing company produces three types of products at two factories. They supply their products at four destinations. The corresponding data are given in **Table - 1.**
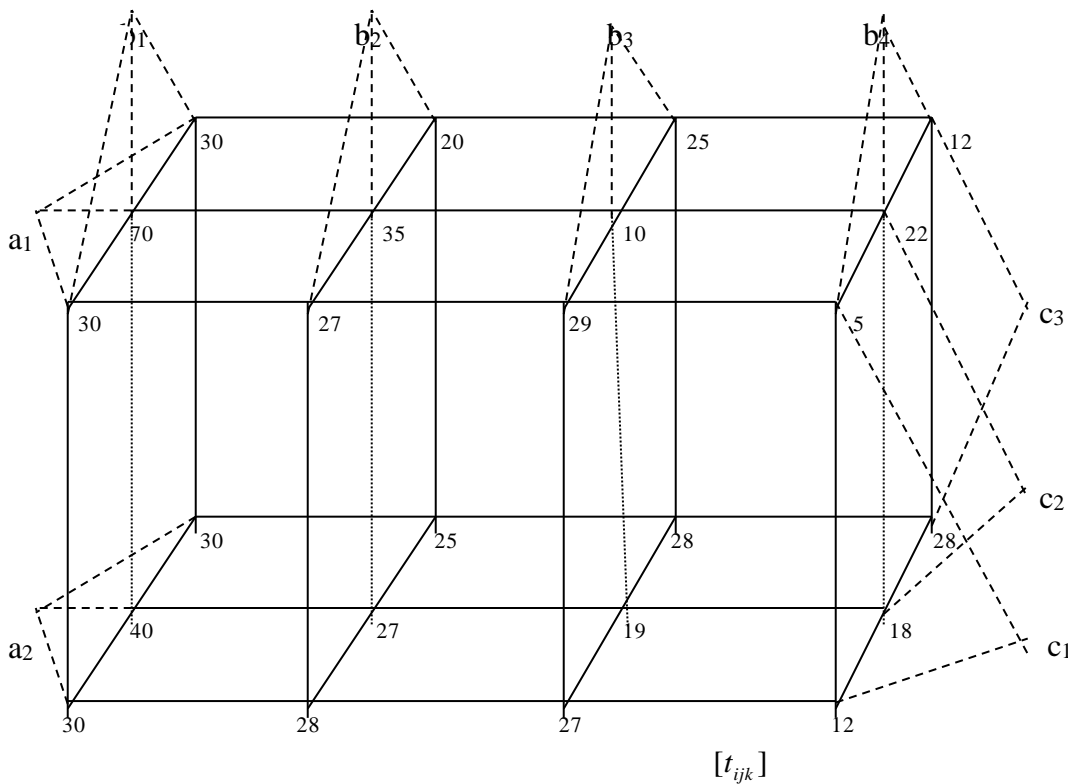
Table – 1

a₁ = 1300 a₂ = 1200	$b_1^* = 250$, b₁ = 300 $b_2^* = 600$, b₂ = 700 $b_3^* = 325$, b₃ = 400 $b_1^* = 250$, b₄ = 500	c₁ = 500 c₂ = 1200 c₃ = 1000

The proposed problem is explained by considering 2 □ 4 □ 3 costs and time minimizing transportation problem, where $[d_{ijk}^1, d_{ijk}^2, r_{ijk}]_{2\times4\times3}$ values and $[t_{ijk}]_{2\times4\times3}$ values are given in **Figure - 1** and **Figure - 2** respectively.
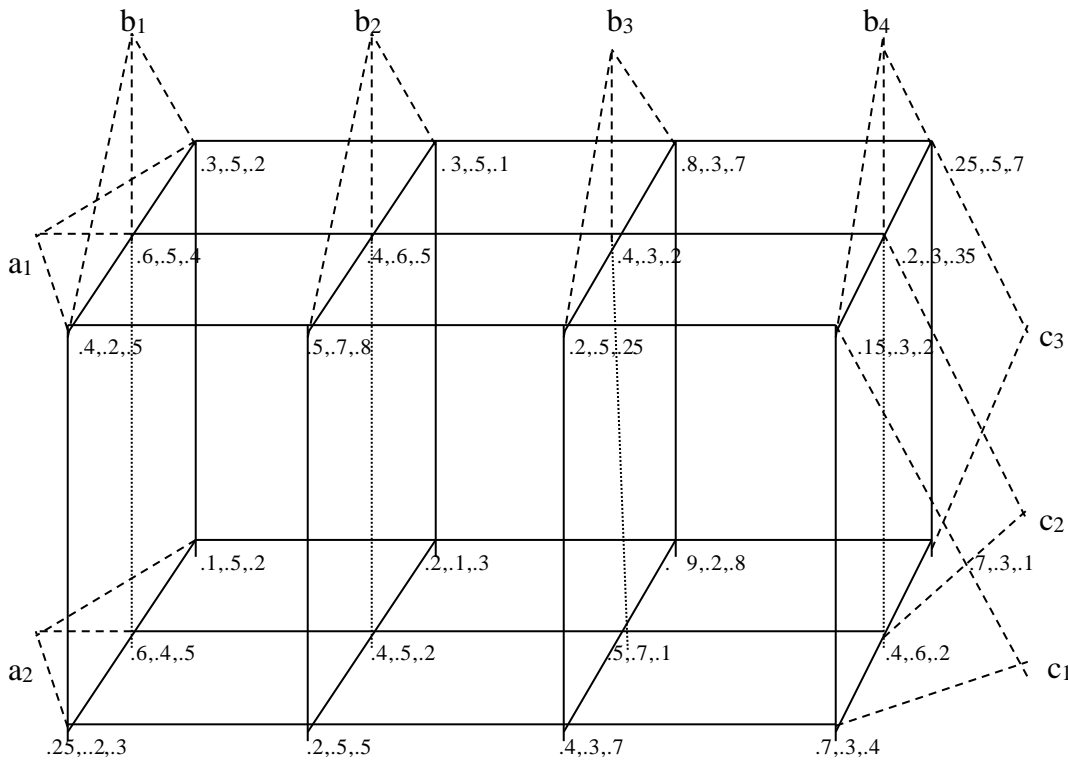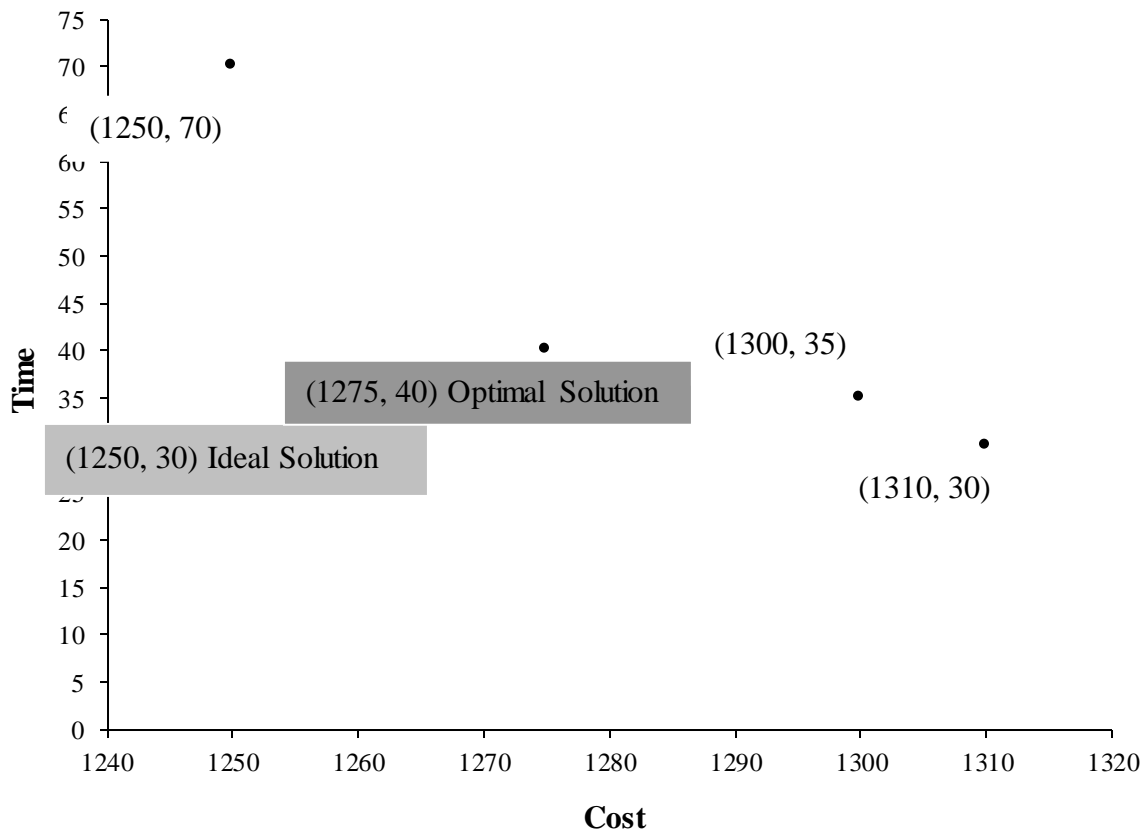
$$[d^1_{ijk}, d^2_{ijk}, r_{ijk}]$$

Figure - 2

$$[t_{ijk}]$$

Four time-cost trade-off pairs (1250, 70), (1275, 40), (1300, 35), (1310, 30) are obtained. The result shows that the ideal solution is (1250, 30). The (D_1) distance of the trade-off pairs from the ideal solution is presented in the **Table - 2.**

Table – 2

Trade-off pairs	Ideal Solution	Distance $(D_1)_r$ between ideal solution and the trade-off pair	$(D_1)_{opt}$	Optimum time-cost trade-off pair
(1250, 70)	(1250, 30)	40	35	(1275, 40)
(1275, 40)		35		
(1300, 35)		55		
(1310, 30)		60		

Time - Cost Graph

So the optimum time-cost trade-off pair is (1275, 40).

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